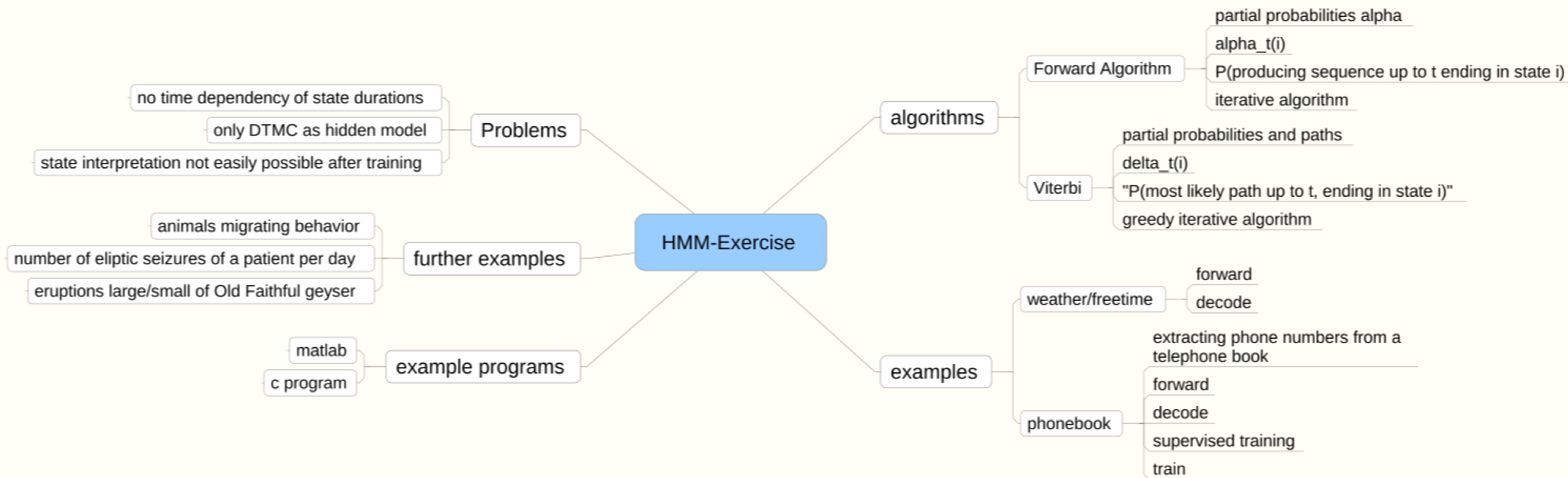
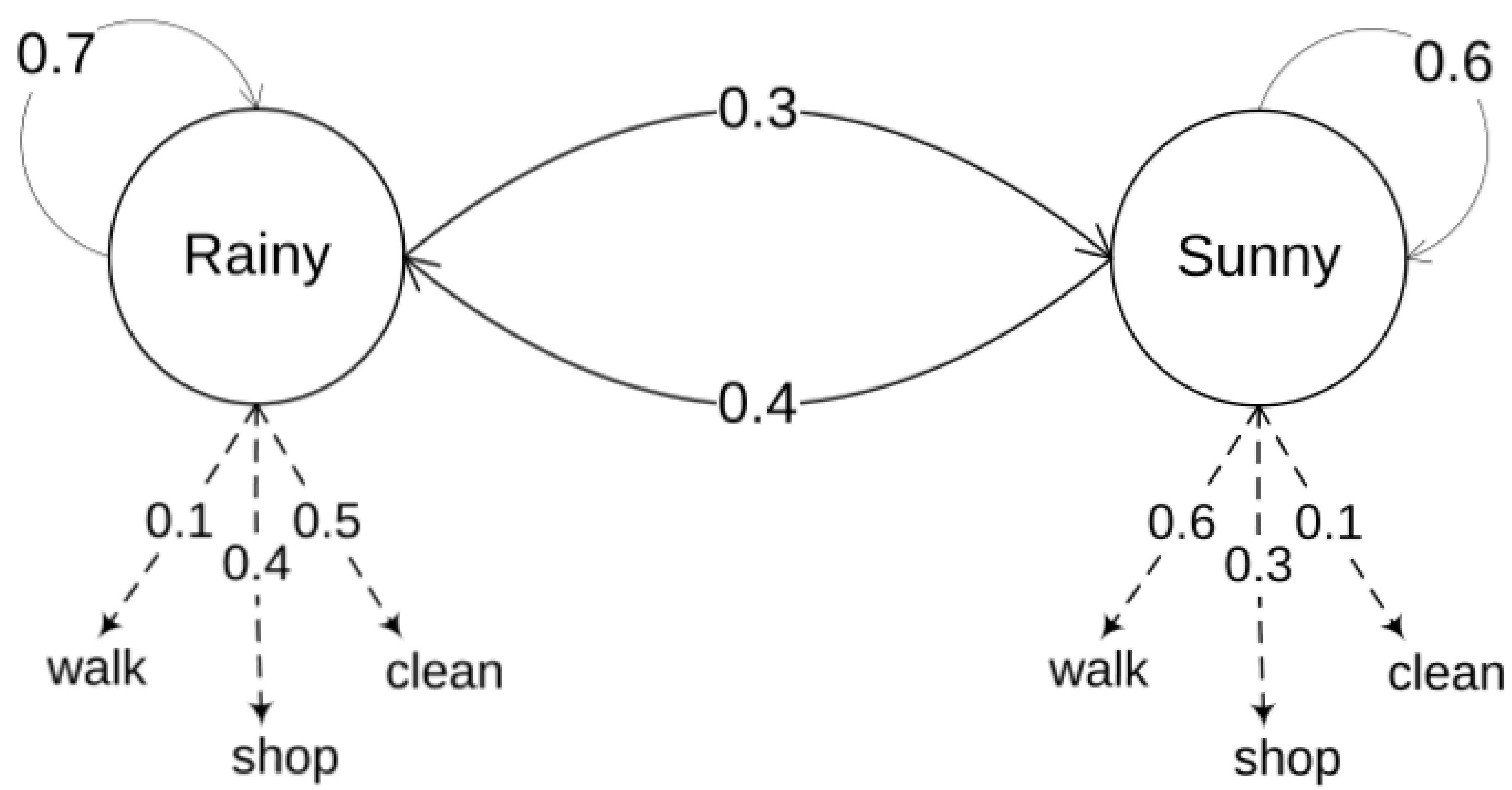

HMM-Exercise
algorithms
Forward Algorithm
partial probabilities alpha
alpha_t(i)
P(producing sequence up to t ending in state i)
iterative algorithm
Viterbi
partial probabilities and paths
delta_t(i)
"P(most likely path up to t, ending in state i)"
greedy iterative algorithm
examples
weather/freetime
forward
decode
phonebook
extracting phone numbers from a telephone book
forward
decode
supervised training
train
Problems
no time dependency of state durations
only DTMC as hidden model
state interpretation not easily possible after training
further examples
animals migrating behavior
number of eliptic seizures of a patient per day
eruptions large/small of Old Faithful geyser
example programs
matlab
c program

ADM - Hidden Markov Models



(S,V,A,B,Π) = ({Rainy,Sunny},{walk,shop,clean},A,B,(0.6,0.4))

$$A = \begin{array}{c} \\ R \\ S \end{array} \begin{array}{cc} R & S \\ \multicolumn{2}{c}{\begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}} \end{array}$$

$$B = \begin{array}{c} \\ R \\ S \end{array} \begin{array}{ccc} w & s & c \\ \multicolumn{3}{c}{\begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}} \end{array}$$

$$P(c\,s\,w \mid RRR)$$

$$= \pi_R \cdot b_{Rc} \cdot a_{RR} \cdot b_{Rs} \cdot a_{RR} \cdot b_{Rw}$$

$$= 0.6 \cdot 0.5 \times 0.7 \times 0.4 \times 0.7 \times 0.1$$

$$= 0.00588$$

$$O = (c\,s\,w)$$

Total Enumeration

$$P(csw/RRR) = \pi_R b_{Rc} a_{RR} b_{Rs} a_{RR} b_{Rw} = 0.6*0.5*0.7*0.4*0.7*0.1 = 0.00588$$
$$P(csw/RRS) = \pi_R b_{Rc} a_{RR} b_{Rs} a_{RS} b_{Sw} = 0.6*0.5*0.7*0.4*0.3*0.6 = 0.01512$$
$$P(csw/RSR) = \pi_R b_{Rc} a_{RS} b_{Ss} a_{SR} b_{Rw} = 0.6*0.5*0.3*0.3*0.4*0.1 = 0.001080$$
$$P(csw/RSS) = \pi_R b_{Rc} a_{RS} b_{Ss} a_{SS} b_{Sw} = 0.6*0.5*0.3*0.3*0.6*0.6 = 0.00972$$
$$P(csw/SRR) = \pi_S b_{Sc} a_{SR} b_{Rs} a_{RR} b_{Rw} = 0.4*0.1*0.4*0.4*0.7*0.1 = 0.000448$$
$$P(csw/SRS) = \pi_S b_{Sc} a_{SR} b_{Rs} a_{RS} b_{Sw} = 0.4*0.1*0.4*0.4*0.3*0.6 = 0.001152$$
$$P(csw/SSR) = \pi_S b_{Sc} a_{SS} b_{Ss} a_{SR} b_{Rw} = 0.4*0.1*0.6*0.3*0.4*0.1 = 0.000288$$
$$P(csw/SSS) = \pi_S b_{Sc} a_{SS} b_{Ss} a_{SS} b_{Sw} = 0.4*0.1*0.6*0.3*0.6*0.6 = 0.002592$$
$$P(csw) = \sum P(csw/xxx) = 0.03628$$

$$Path = \arg\max_{xxx} \{P(csw/xxx)\} = RRS$$

$$O, \lambda \rightarrow P(O|\lambda)$$

$$\alpha_t(i) = P(o_1 o_2 \dots o_t, q_t = s_i | \lambda)$$

$$\alpha_1(i) = \pi_i b_i(o_1)$$

$t = 2 \dots T$

$$\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t)$$

$$P(O|\lambda) = \sum_{i=1}^{N} \alpha_T(i)$$

$$\alpha_1(R) = \pi_R \cdot b_{Rc} = 0.6 \times 0.5 = 0.3$$

$$\alpha_1(S) = \pi_S \times b_{Sc} = 0.4 \times 0.1 = 0.04$$

$$\alpha_2(R) = \alpha_1(R) \cdot a_{RR} \cdot b_{Rs} + \alpha_1(S) \cdot a_{SR} \cdot b_{Rs}$$

$$\alpha_2(S) = \alpha_1(R) \cdot a_{RS} \, b_{Ss} + \alpha_1(S) a_{SS} \cdot b_{Ss}$$

$$\alpha_3(R) = \alpha_2(R) a_{RR} b_{Rw} + \alpha_2(S) a_{SR} \, b_{Rw}$$

$$\alpha_3(S) = \alpha_2(R) a_{RS} b_{Sw} + \alpha_2(S) a_{SS} \, b_{Sw}$$

$$\alpha_3(R) + \alpha_3(S) = P(csw \mid \lambda)$$

Evaluation

Forward Algorithm

$$\alpha_1(R) = \pi_R b_{Rc} = 0.6 * 0.5 = 0.3$$

$$\alpha_1(S) = \pi_S b_{Sc} = 0.4 * 0.1 = 0.04$$

$$\alpha_2(R) = \alpha_1(R) a_{RR} b_{Rs} + \alpha_1(S) a_{SR} b_{Rs} = 0.3 * 0.7 * 0.4 + 0.04 * 0.4 * 0.4 = 0.0904$$

$$\alpha_2(S) = \alpha_1(R) a_{RS} b_{Ss} + \alpha_1(S) a_{SS} b_{Ss} = 0.3 * 0.3 * 0.3 + 0.04 * 0.6 * 0.3 = 0.0342$$

$$\alpha_3(R) = \alpha_2(R) a_{RR} b_{Rw} + \alpha_2(S) a_{SR} b_{Rw} = 0.0904 * 0.7 * 0.1 + 0.0342 * 0.4 * 0.1 = 0.007696$$

$$\alpha_3(S) = \alpha_2(R) a_{RS} b_{Sw} + \alpha_2(S) a_{SS} b_{Sw} = 0.0904 * 0.3 * 0.6 + 0.0342 * 0.6 * 0.6 = 0.028584$$

$$P(csw) = \alpha_3(R) + \alpha_3(S) = 0.03628$$

BF $O(N^T)$ For $O(N \times T)$

$$O, \lambda \rightarrow Q$$

$$\delta_t(i) = \max_{q_1 \dots} P(q_1 \dots q_{t-1}, q_t = s_i, o_1 \dots o_t \mid \lambda)$$

$$\delta_1(i) = \pi_i \, b_i(o_1)$$

$t = 2 \dots T$

$$\delta_t(j) = \max_i \left[\delta_{t-1}(i) \, a_{ij} \right] b_j(o_t)$$

$$P(O \mid Q, \lambda) = \max_i (\delta_T(i))$$

backtracking $\rightarrow Q$

csw

$$\delta_1(R) = \pi_R \cdot b_{RC} = 0.3$$

$$\delta_1(S) = \pi_S \cdot b_{SC} = 0.04$$

$$\delta_2(R) = \max\left[\underline{\delta_1(R) a_{RR} b_{RS}}, \delta_1(S) a_{SR} b_{RS}\right]$$

$$\delta_2(S) = \max\left[\underline{\delta_1(R) a_{RS} b_{SS}}, \delta_1(S) a_{SS} b_{SS}\right]$$

$$\delta_3(R) = \max\left[\delta_2(R) a_{RR} b_{RW}, \delta_2(S) a_{SR} b_{RW}\right]$$

$$\delta_3(S) = \max\left[\delta_2(R) a_{RS} b_{SW}, \delta_2(S) a_{RS} b_{SW}\right]$$

$$\max\left[\delta_2(R), \delta_3(S)\right]$$

Decoding

Viterbi Algorithm

$$\delta_1(R) = \pi_R b_{Rc} = 0.6 * 0.5 = 0.3$$

$$\delta_1(S) = \pi_S b_{Sc} = 0.4 * 0.1 = 0.04$$

$$\delta_2(R) = \max\{\delta_1(R) a_{RR} b_{Rs}, \delta_1(S) a_{SR} b_{Rs}\} = \max\{0.084, 0.0064\} = 0.084$$

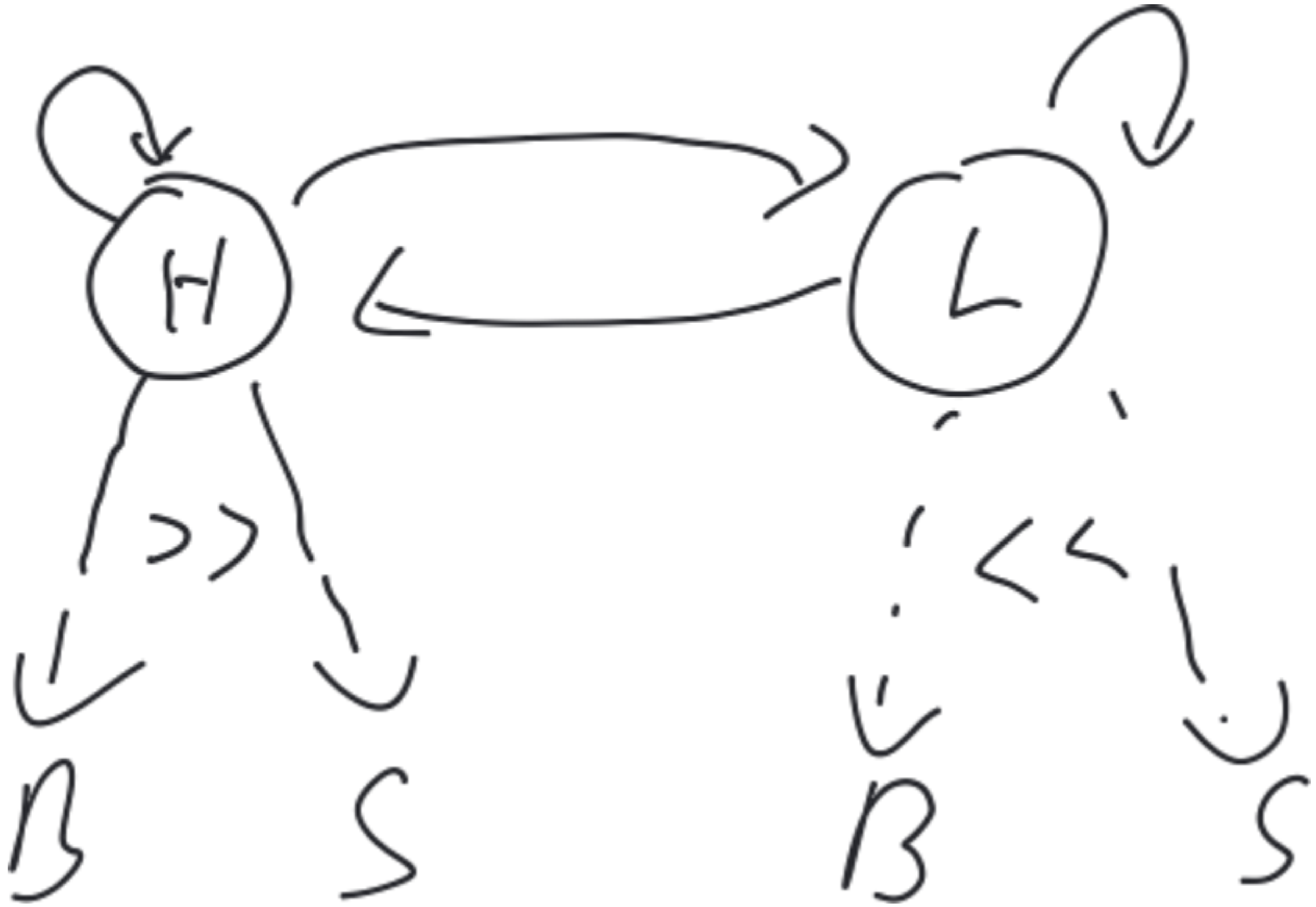
$$\delta_2(S) = \max\{\delta_1(R) a_{RS} b_{Ss}, \delta_1(S) a_{SS} b_{Ss}\} = \max\{0.027, 0.0072\} = 0.027$$

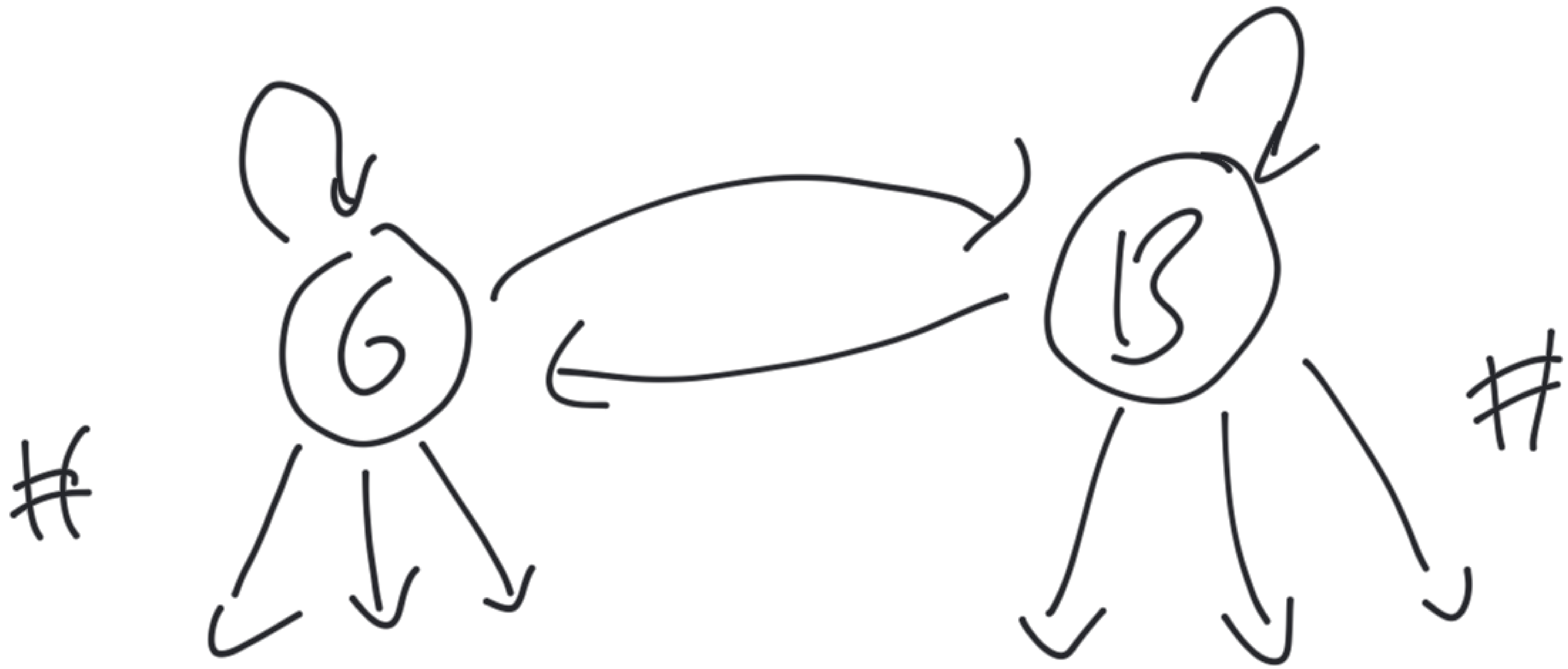
$$\delta_3(R) = \max\{\delta_2(R) a_{RR} b_{Rw}, \delta_2(S) a_{SR} b_{Rw}\} = \max\{0.00588, 0.00108\} = 0.00588$$

$$\delta_3(S) = \max\{\delta_2(R) a_{RS} b_{Sw}, \delta_2(S) a_{SS} b_{Sw}\} = \max\{0.01512, 0.00972\} = 0.01512$$

$$\max\{\delta_3(R), \delta_3(S)\} = 0.01512 = P(csw / RRS)$$

RRS


H
L
>>
<<
B
S
B
S


#
6
13
#

rating migration