GSPNs and CTMCs

- History
  - 1960er - Carl Adam Petri: S/T-Nets
  - contain only places, transitions, arcs and tokens
  - 1980/81 Molloy: Stochastic Petri Nets
  - contain timed transitions with exponential distributions
  - every SPN describes a CTMC!
  - no multiplicities, guard functions inhibitors...
- State Space
  - marking - number of tokens in every place of the SPN
  - state space = set of all reachable markings
  - dependent on the initial marking
  - example: 2 places / 2 transitions - size n+1
  - example: 3 places / 4 transitions. - size ~ (n+1)^2/2
  - polynomial growth of size of state space
- Reachability Graph
  - state space plus state transitions
  - can depend on initial marking
  - arcs = SPN transitions
  - if real SPN --> RG=CTMC
  - SPN is more compact than CTMC
  - SPNs widely used, since CTMCs are easy to solve
- Generalized Stochastic Petri Nets
  - removing vanishing states --> CTMC
  - for exp: rate * probability = new rate
  - tangible state vs. vanishing state
  - queue-2 server example
  - extends SPN by immediate transitions
  - 1984: Marsan, Balbo, Conte
- Measures for GSPNs
  - P(transition active
  - transition throughput
  - Avg #Tokens in place
  - P(Place is empty)
  - Queuing example
- dual view
  - time discrete: stochastic
  - time continuous: deterministic
  - linked via exp distribution
  - two state A->B example

GSPNs.mmap - 30.04.2013 -

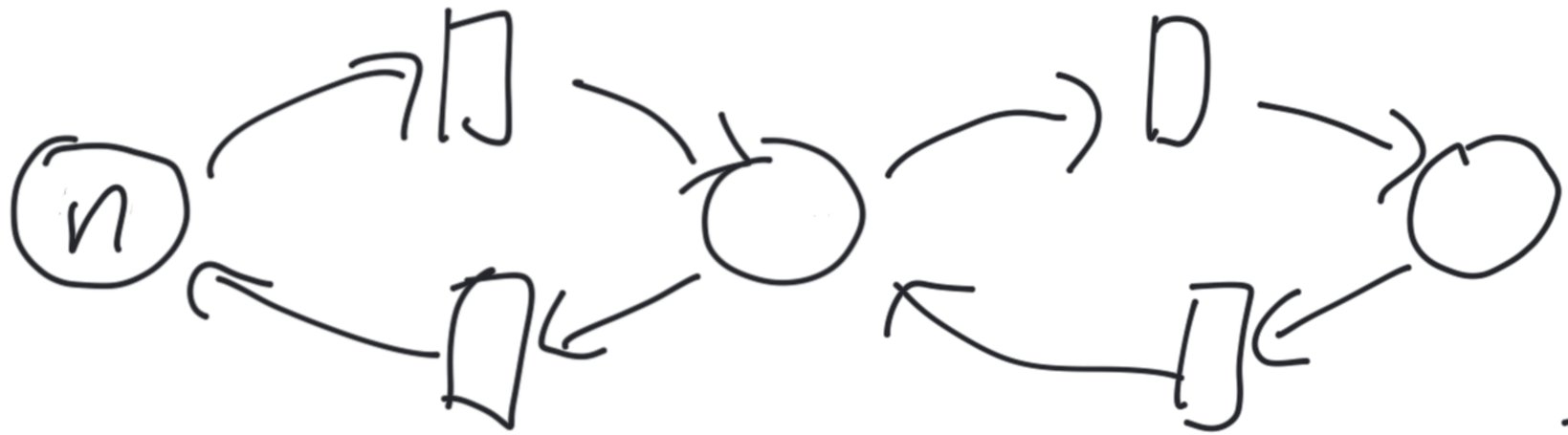
1960 S/T

1980/81 SPN

$\sim Exp$

→ n+1 states

marking (2,0)

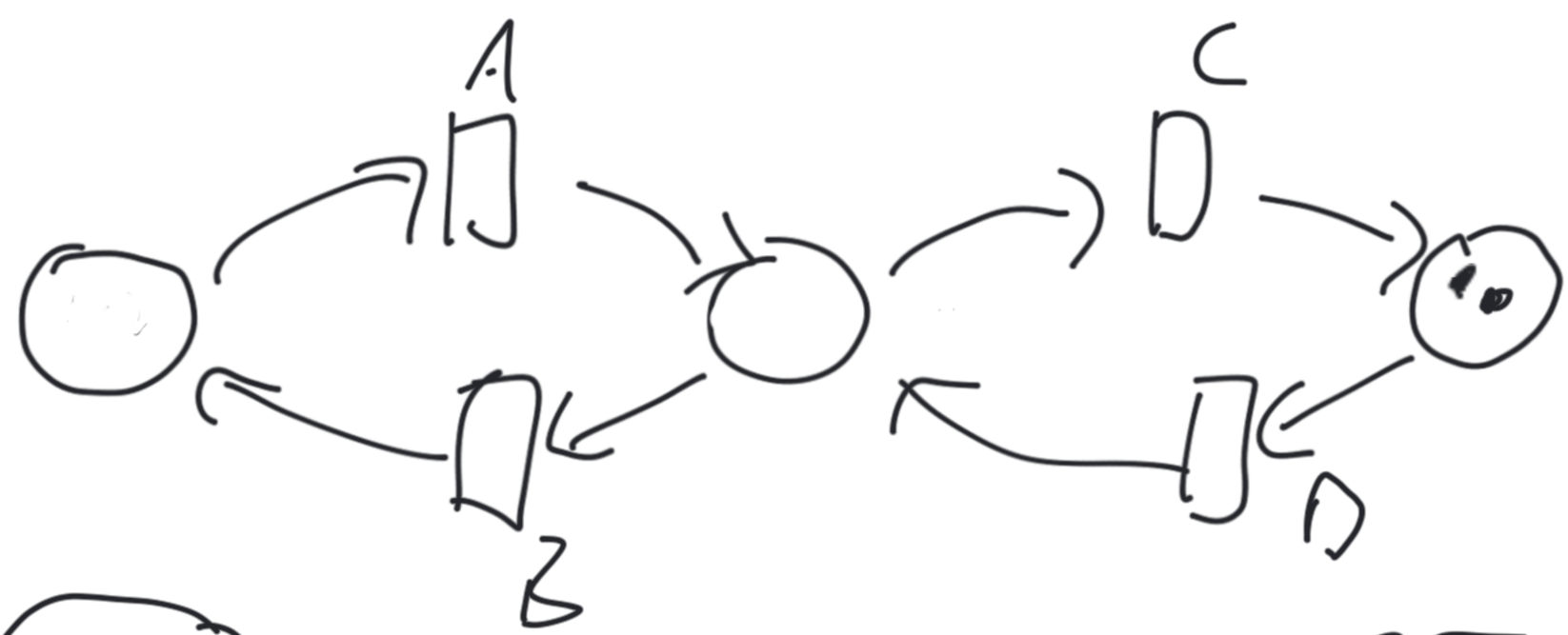


$(n,0,0)$

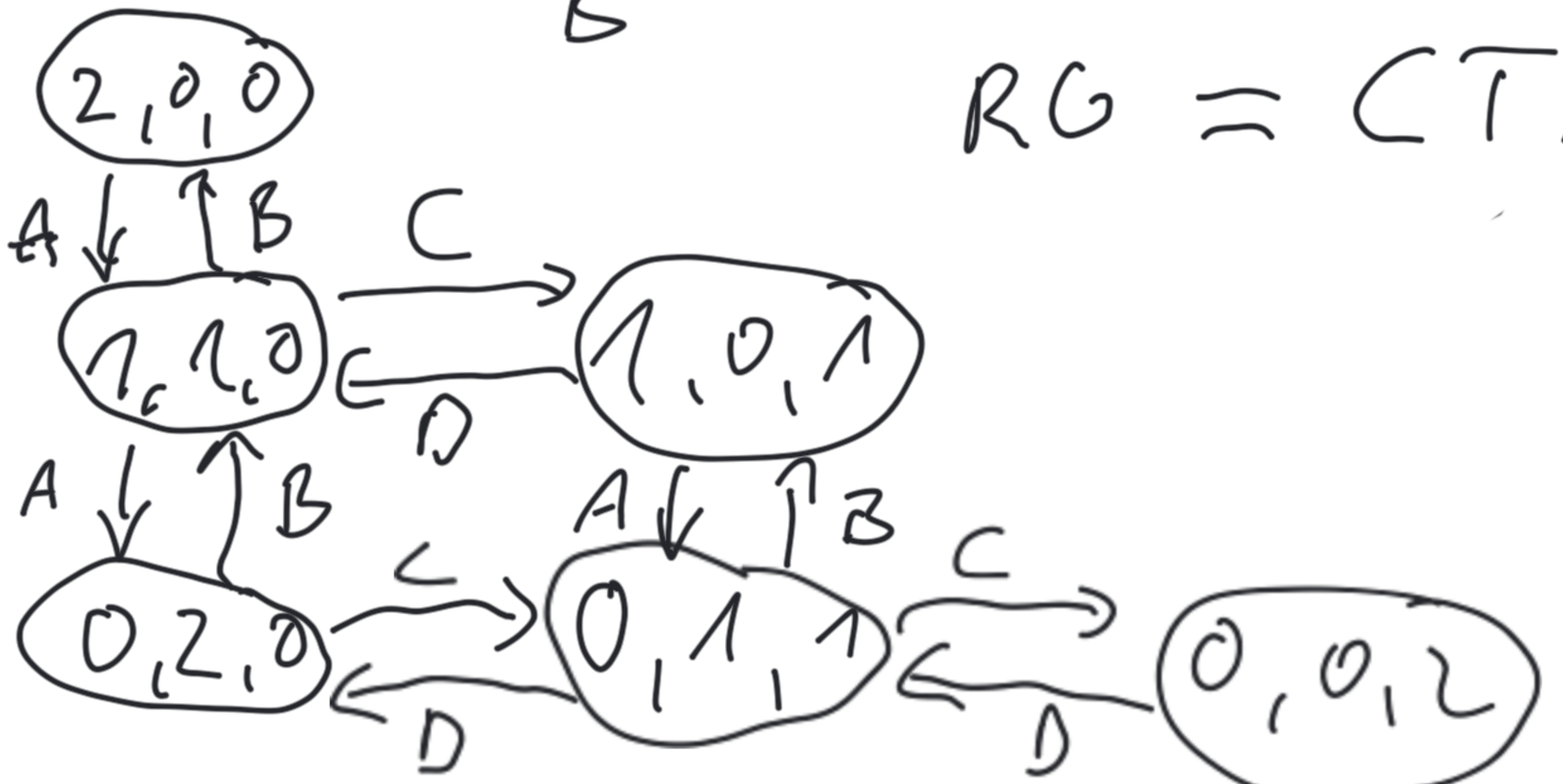
$(n-1,1,0)(n\cdot 1,0,1)$ $\simeq$ $\frac{(n+1)^2}{2}$

$\vdots$ $\ddots$

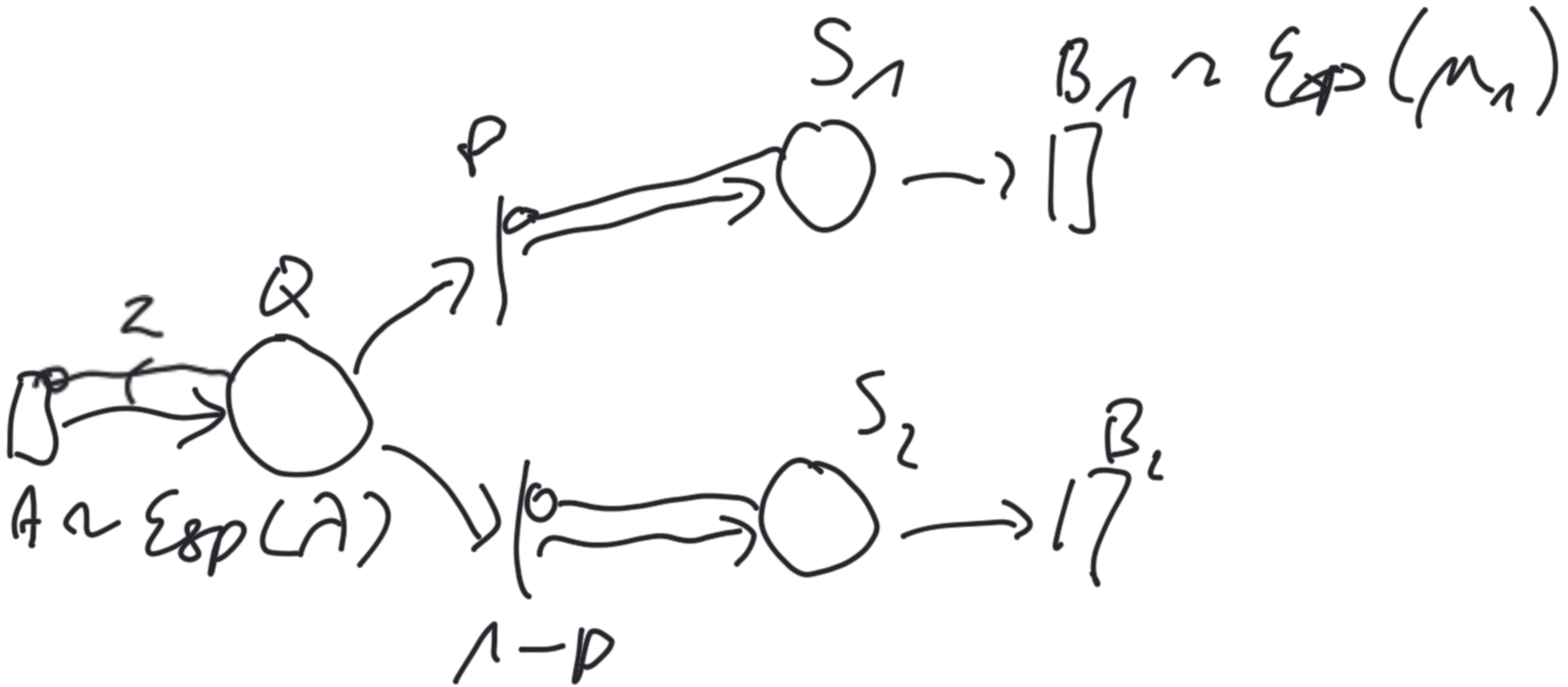
$(0,n,0)(0,n-1,1)\cdots\cdots(0,0,n)$

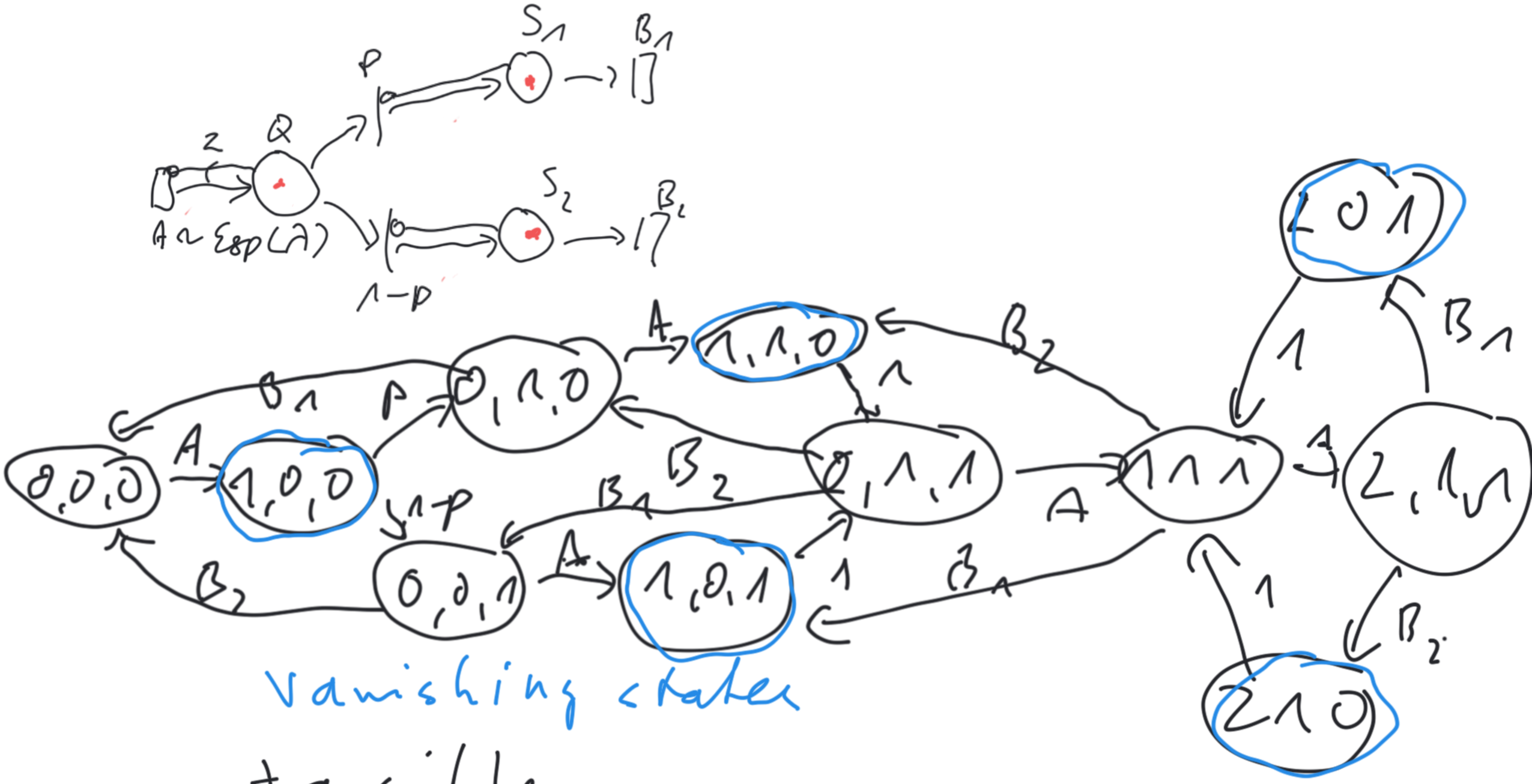


RG ≅ CTMC



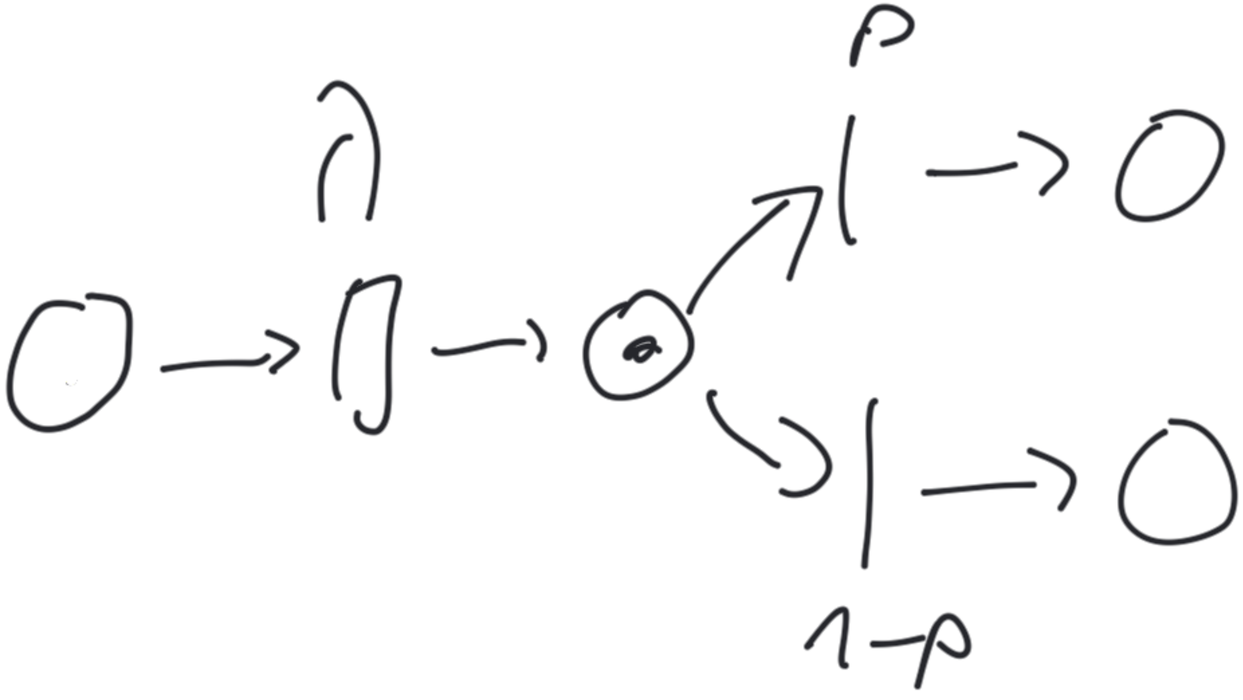
1984 SPN + Imm T = GSPN

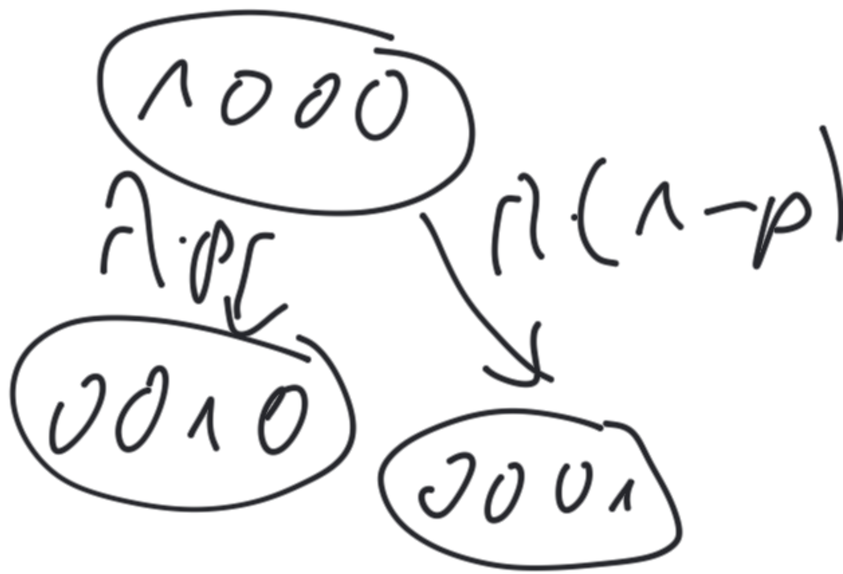


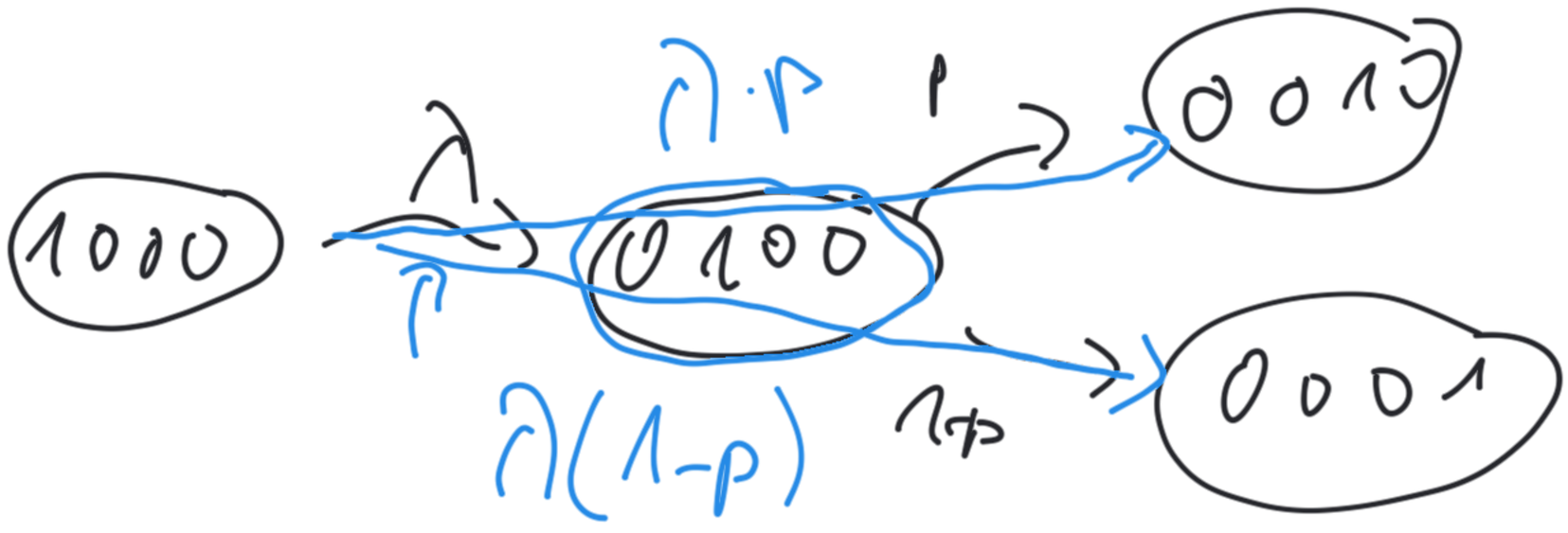


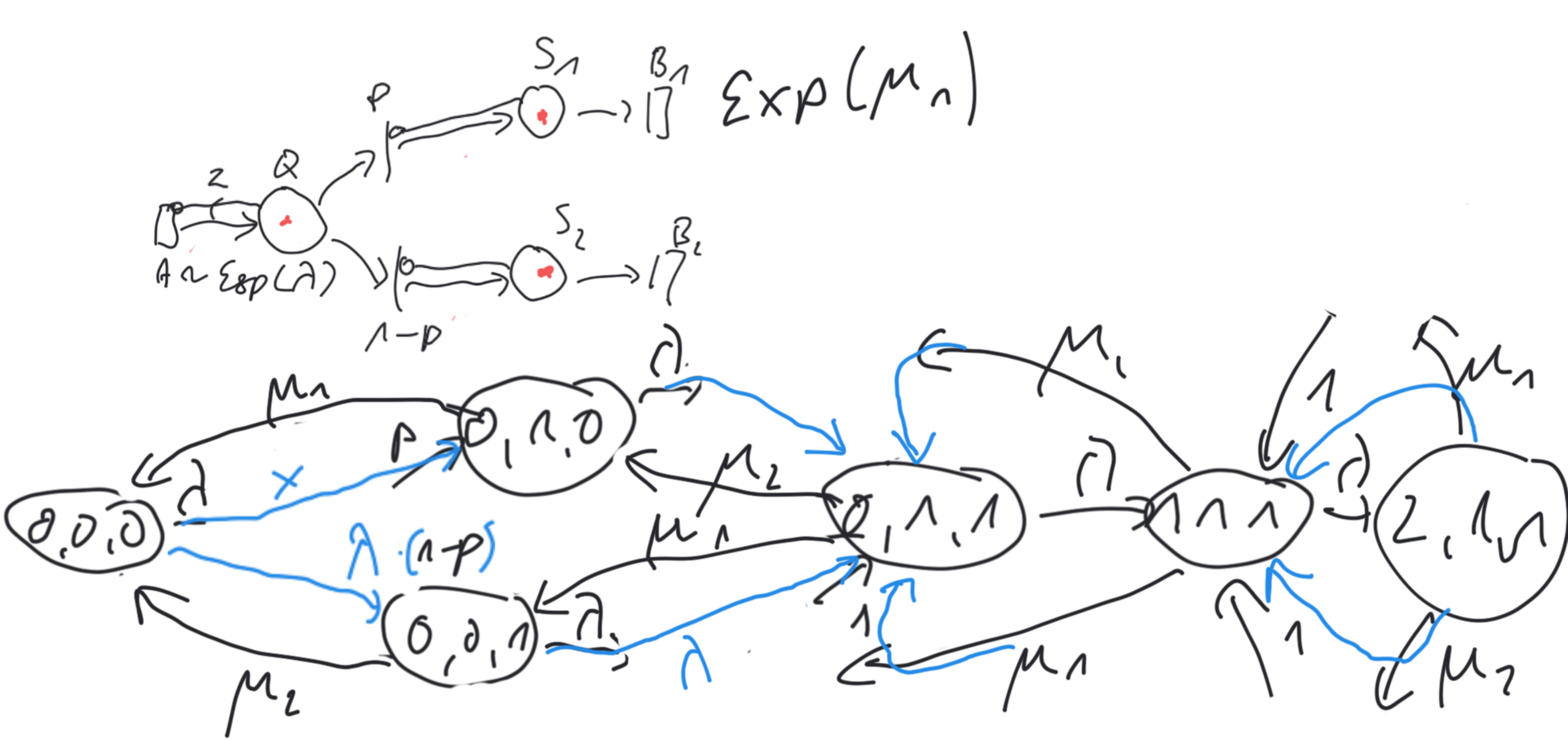
vanishing states

tangible

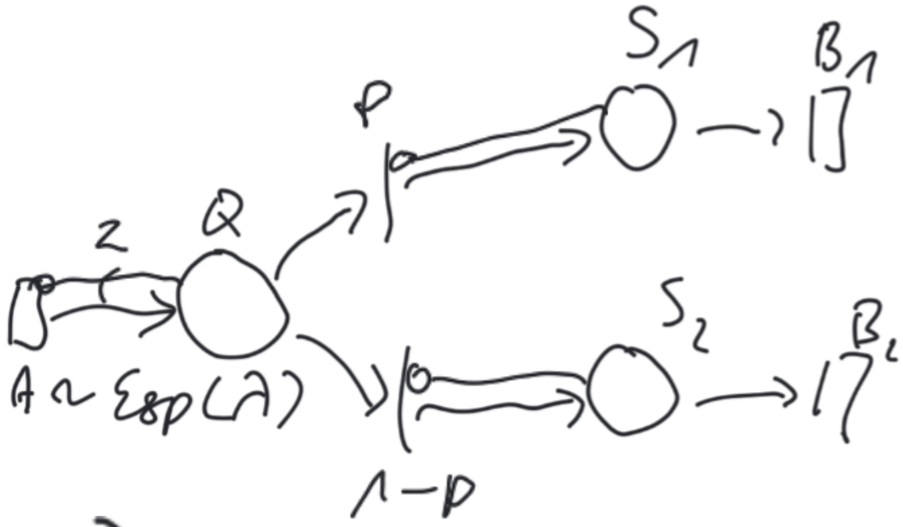

λ
p
1
0
1-p

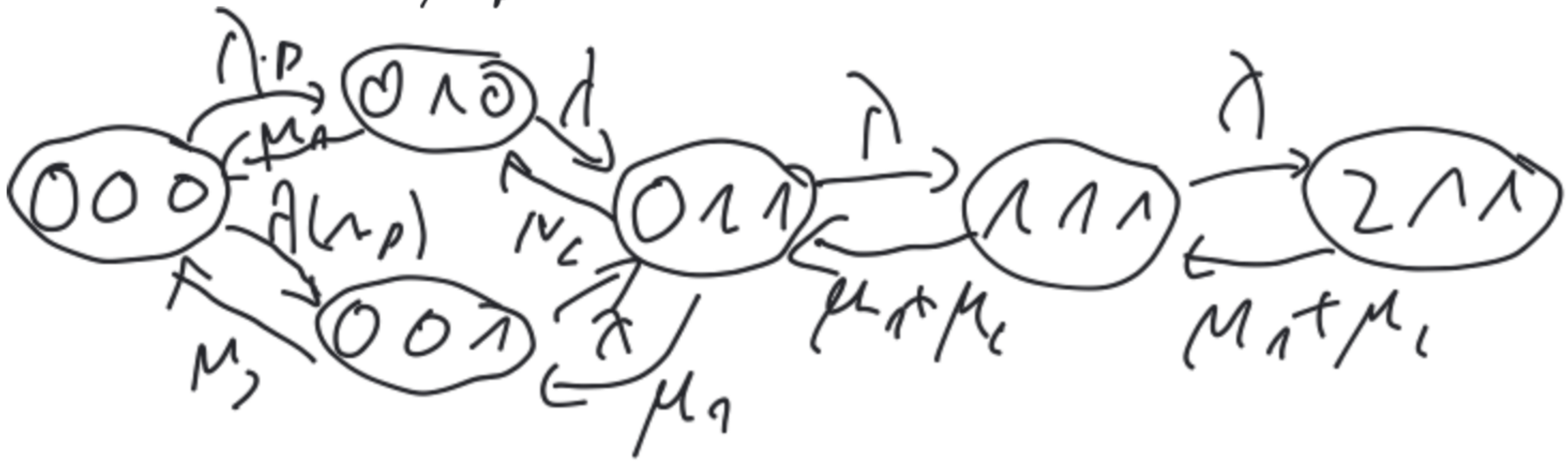

1000
λ·p
(1-p)
0010
0001


1000
λ
λ·p
p
0100
r
λ(1-p)
1-p
0001
0001

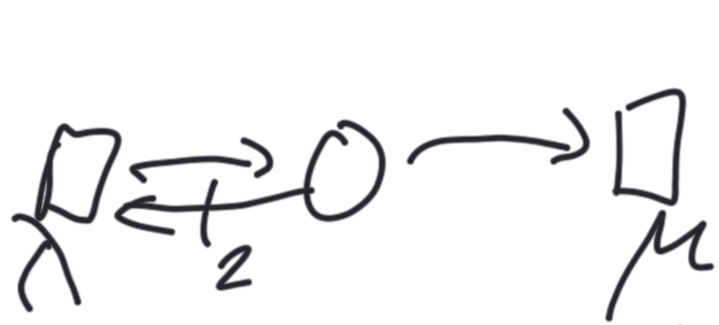


Exp(μ1)
S1
B1
P
Q
z
A ~ Exp(λ)
S2
B2
1-p
0,0,0
0,1,0
0,0,1
2,1,1
1,1,1
2,1,1
λ
μ1
μ2
p
λ·(1-p)
1





$\Rightarrow$ CTMC



(π_0, π_1, π_2)

1) P(place is empty) $= \pi_0$

$\Sigma \pi_i$ (when P empty)

2) Avg # tokens in Place P $= \pi_1 \cdot 1 + \pi_2 \cdot 2$

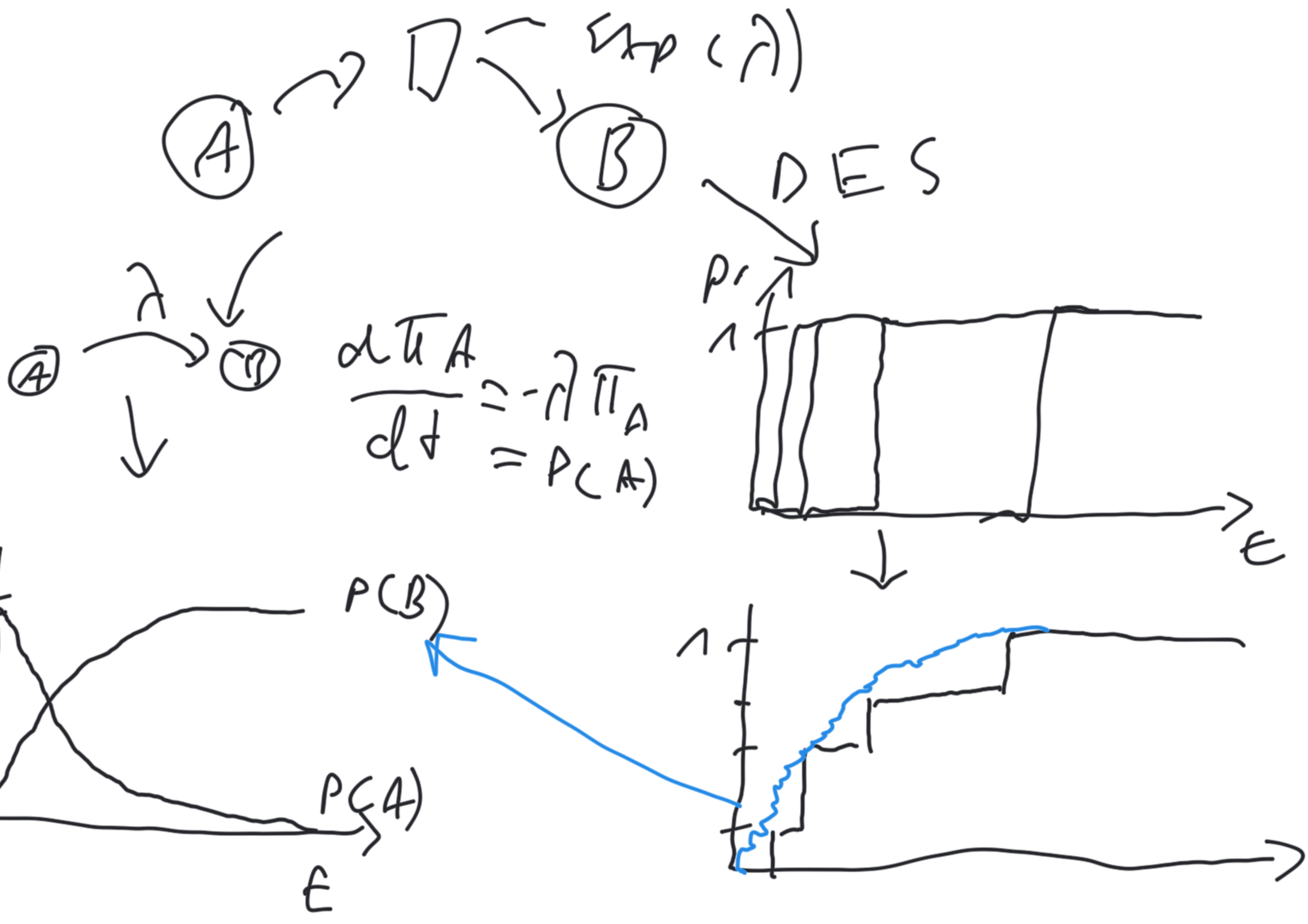
$\Sigma \pi_i \cdot \#t_{i,P}$

3) P(T is active) $= \pi_1 + \pi_2$

$\Sigma \pi_i$ (T is active in i)

$P(T\text{ alive}) \times \mu_T$

4) Throughput T $= (\pi_1 + \pi_2) \cdot \mu$


A
B
Exp (λ)
λ
A
B
dπA/dt = -λ πA
= P(A)
DES
P
t
ε
t
P(B)
P(A)
ε
t