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UNIVERSITÄT
MAGDEBURG

INF

FAKULTÄT FÜR
INFORMATIK

Applied Discrete Modelling

GSPNs & CTMCs - Examples



Example: The Hairdresser

One person is at a beauty salon

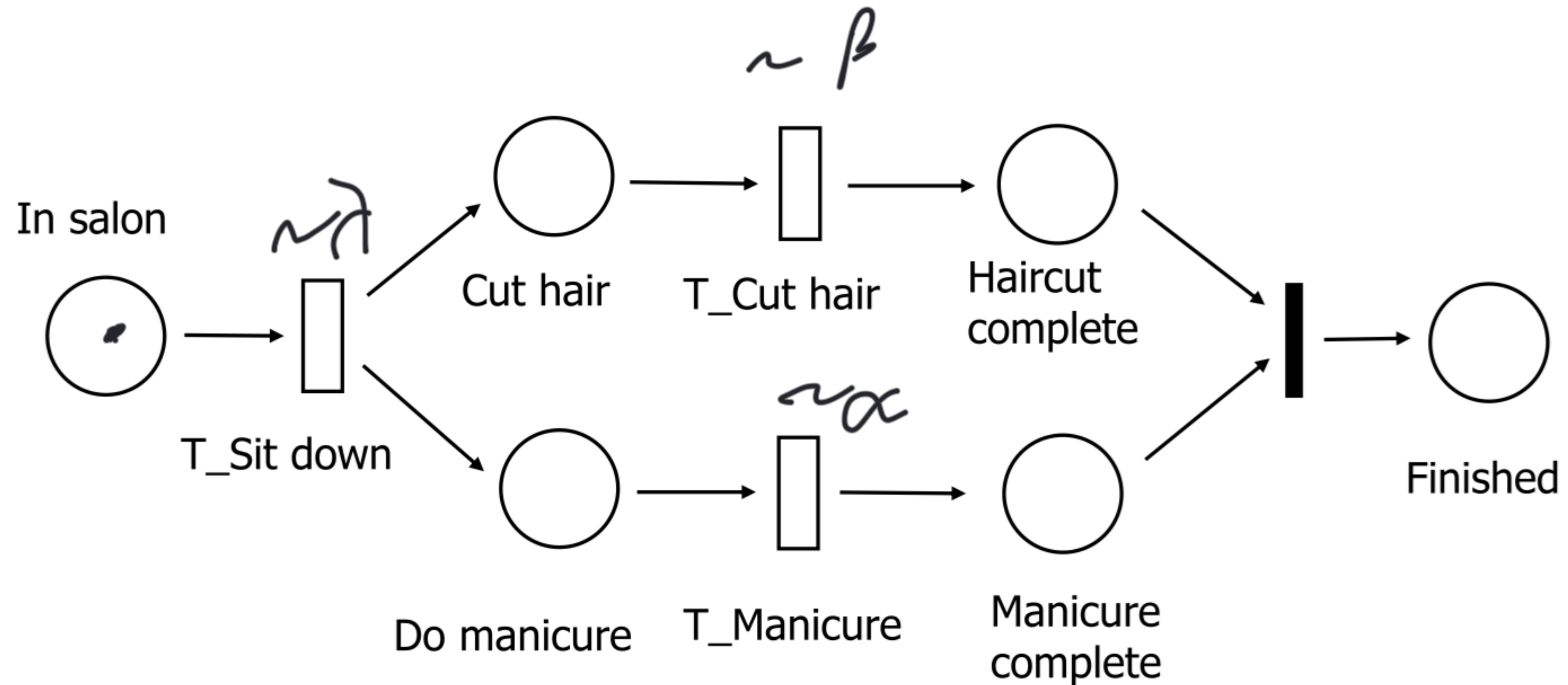
It takes a certain amount of time to be seated $\sim \alpha$

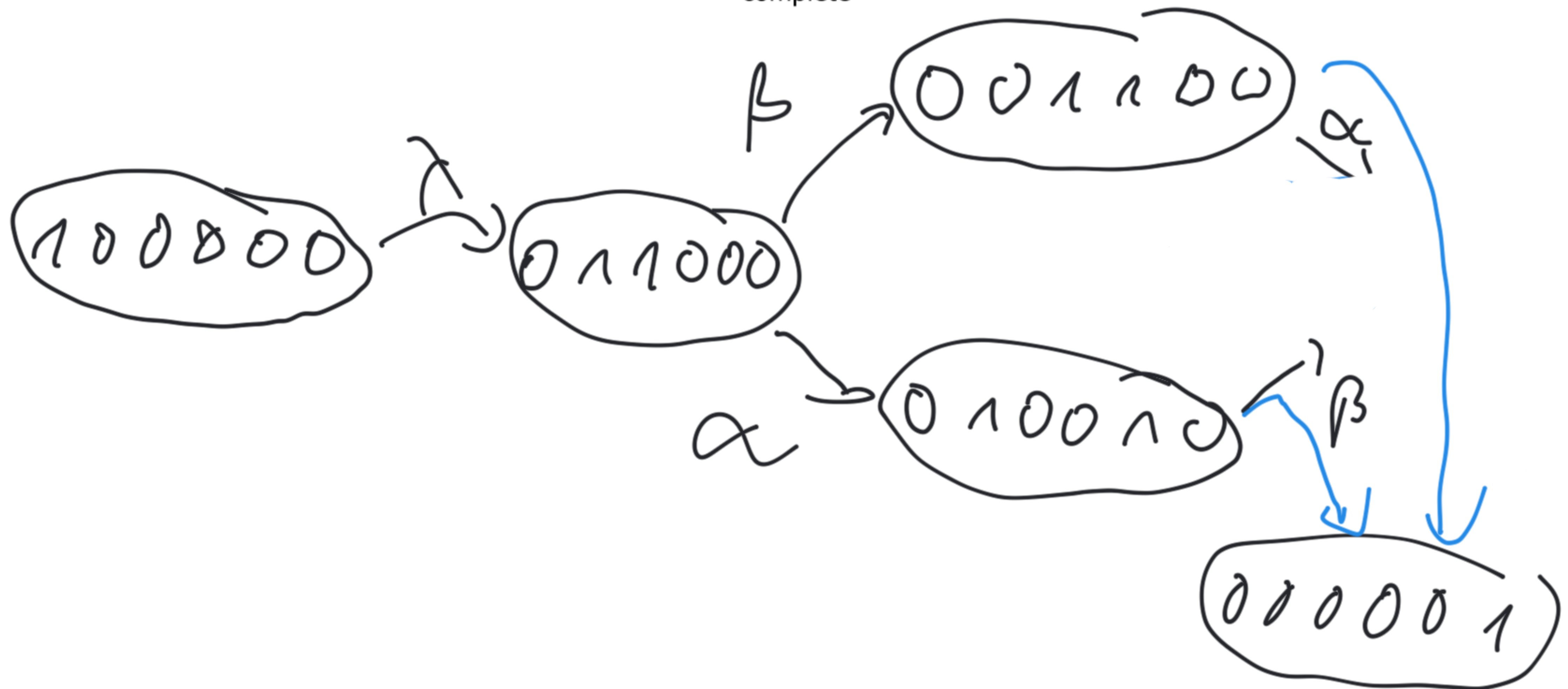
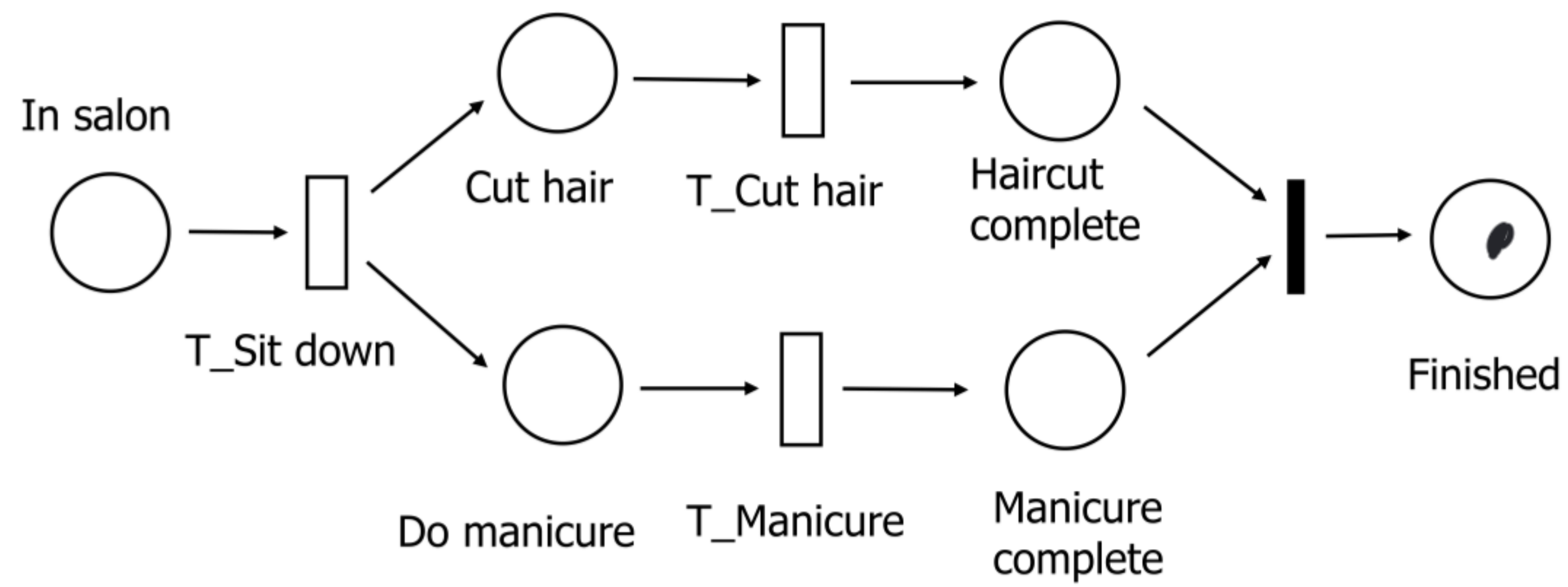
Then the manicure and haircut can start

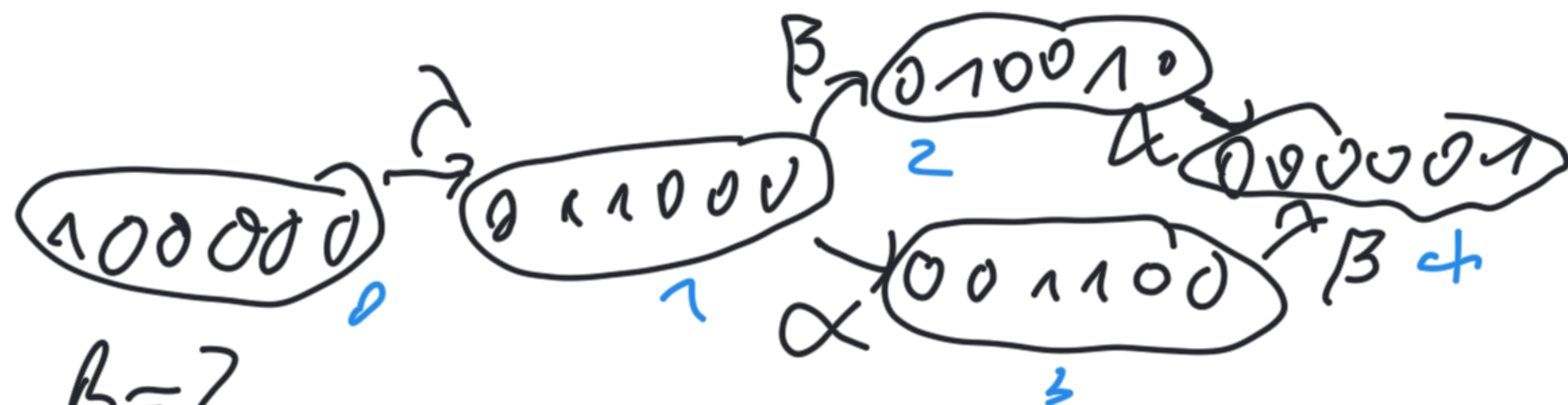
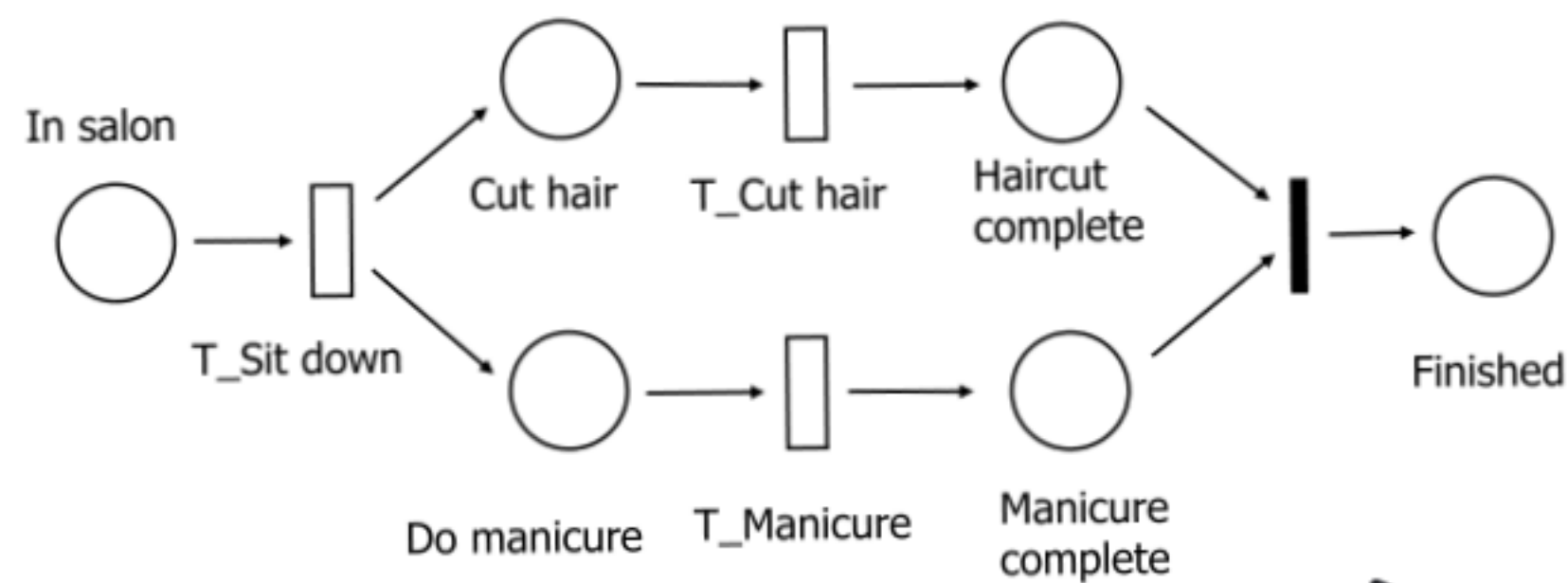
Both can happen at the same time and can have different durations $\sim \beta$
 α

Only if both manicure and haircut are done, is the visit at the salon finished

Example: The Hairdresser







$$\eta = 9 \quad \alpha = 3 \quad \beta = 2$$

$$\begin{bmatrix} -\eta & \eta & 0 & 0 & 0 \\ 0 & \alpha - \beta & \beta & \alpha & 0 \\ 0 & 0 & -\alpha & \alpha & \alpha \\ 0 & 0 & 0 & -\beta & \beta \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -9 & 9 & 0 & 0 & 0 \\ 0 & -5 & 2 & 3 & 0 \\ 0 & 0 & -3 & 0 & 3 \\ 0 & 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Delta < \frac{1}{9}$$

$\frac{1}{c_0}$

Example: The Bar

3 people are sitting in a bar

The time until one of them orders their next drink is
exponentially distributed $\sim A$

If the barman is free, he can attend to the request

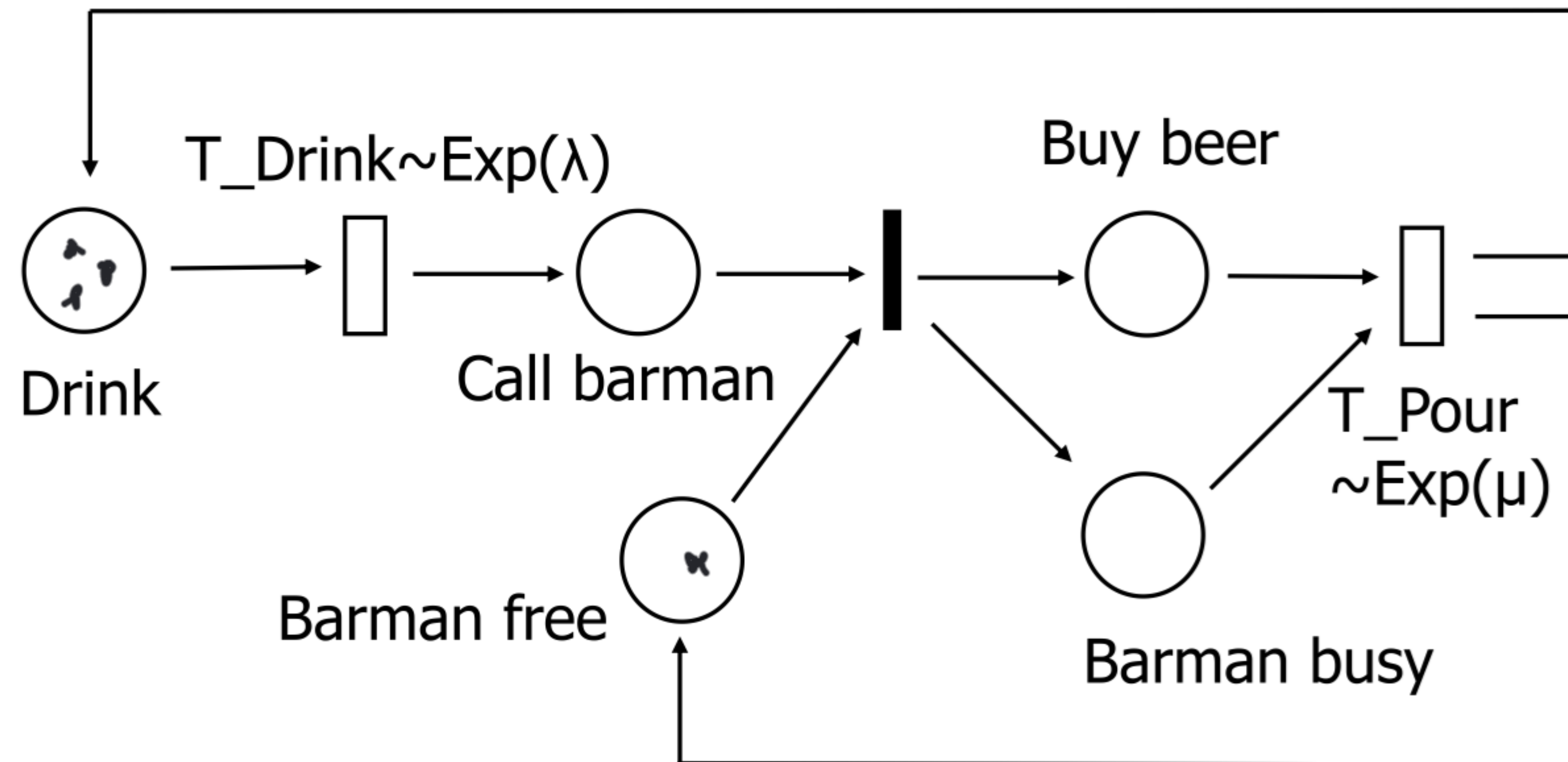
Preparing the drink and serving it takes an exponentially
distributed amount of time $\sim \mu$

After that the customer is happy again, and the barman free

Example: The Bar

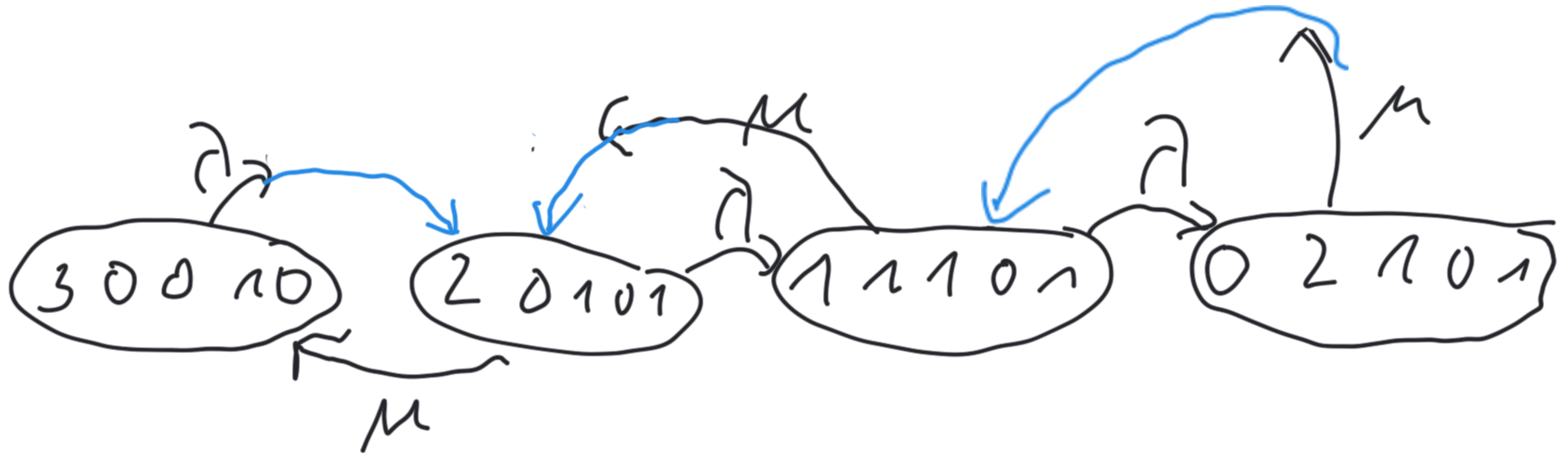
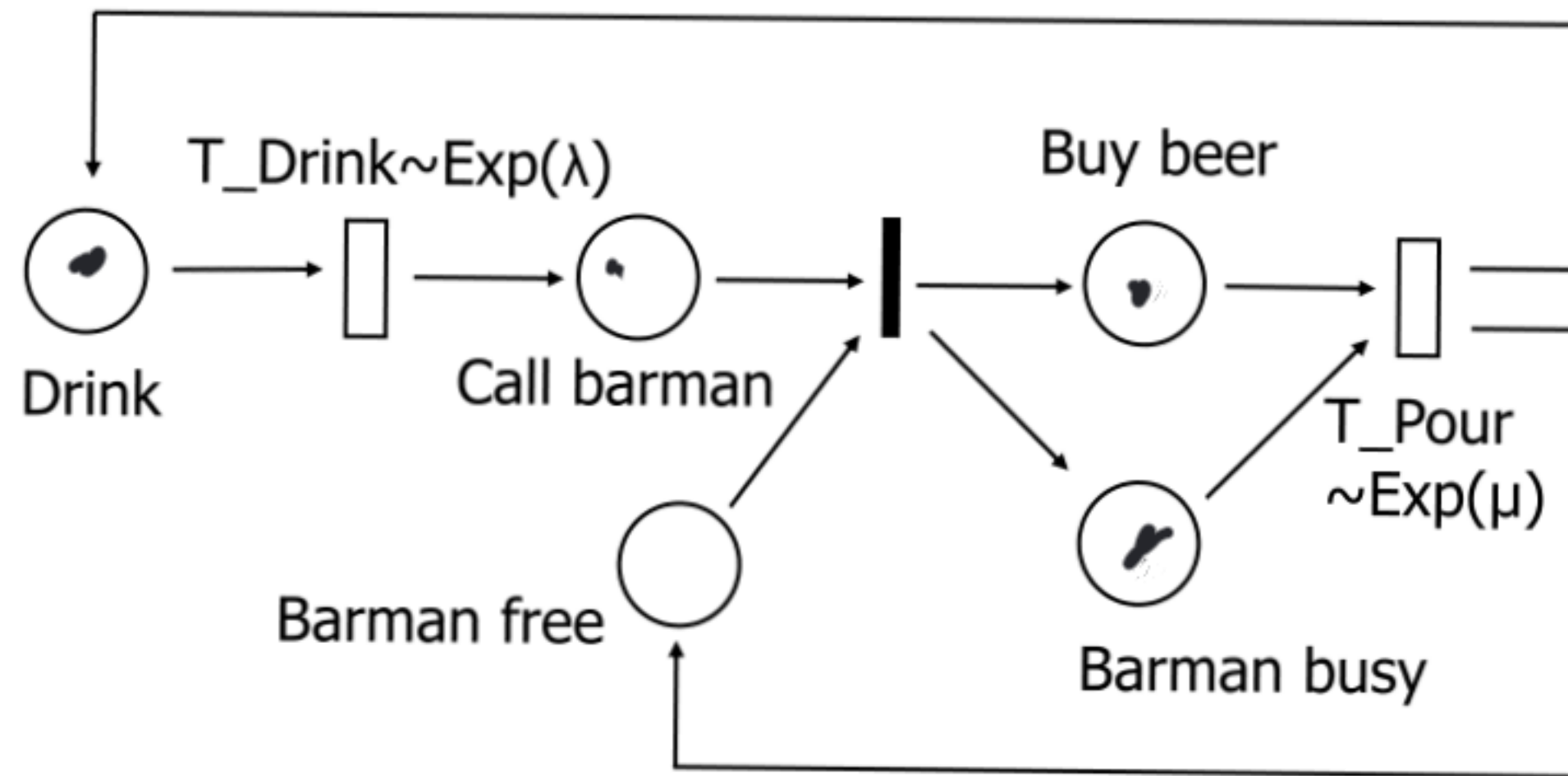
P (no customer is waiting)

Average number of waiting customers?



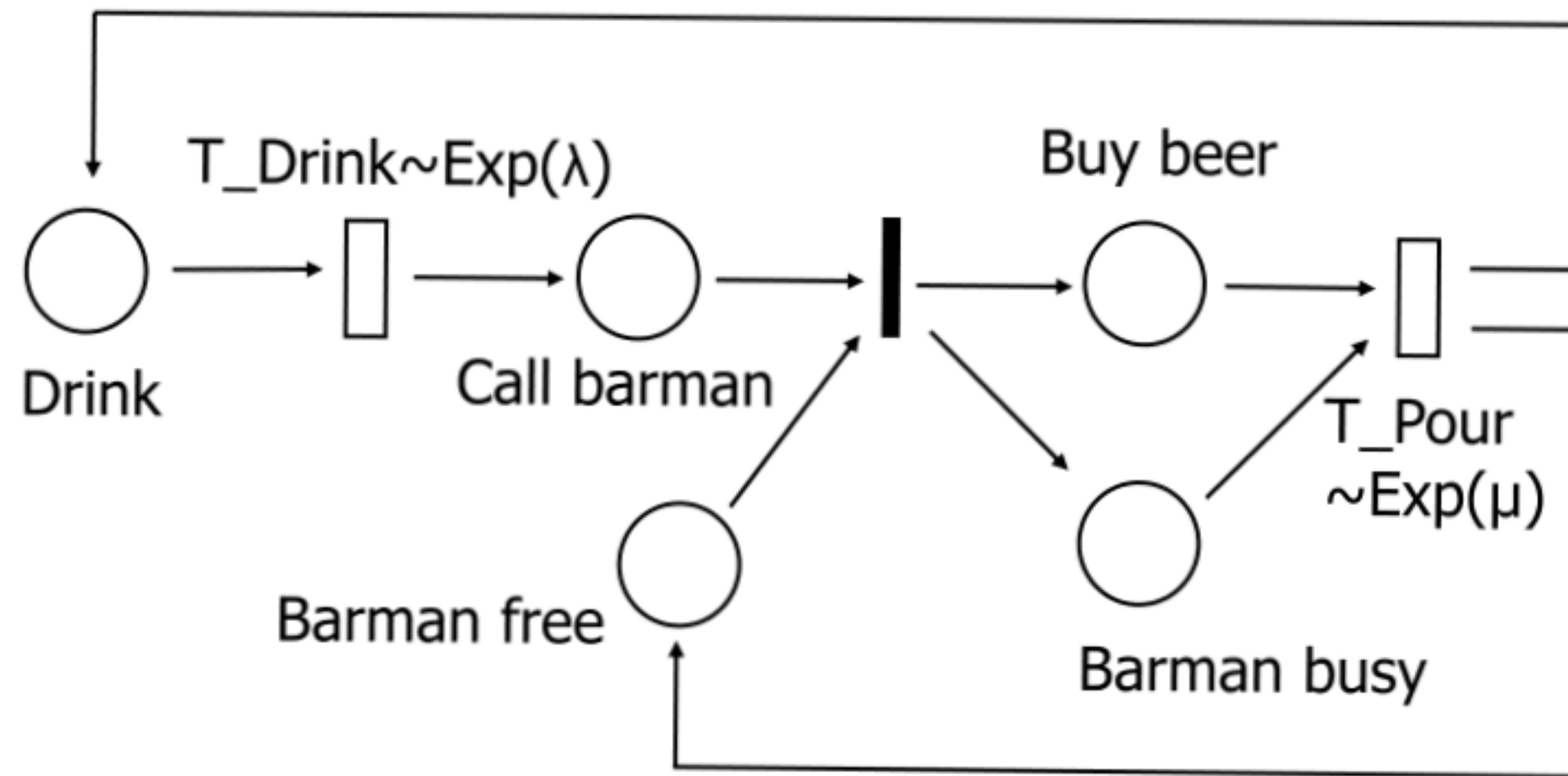
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Average number of waiting customers?



P (no customer is waiting)

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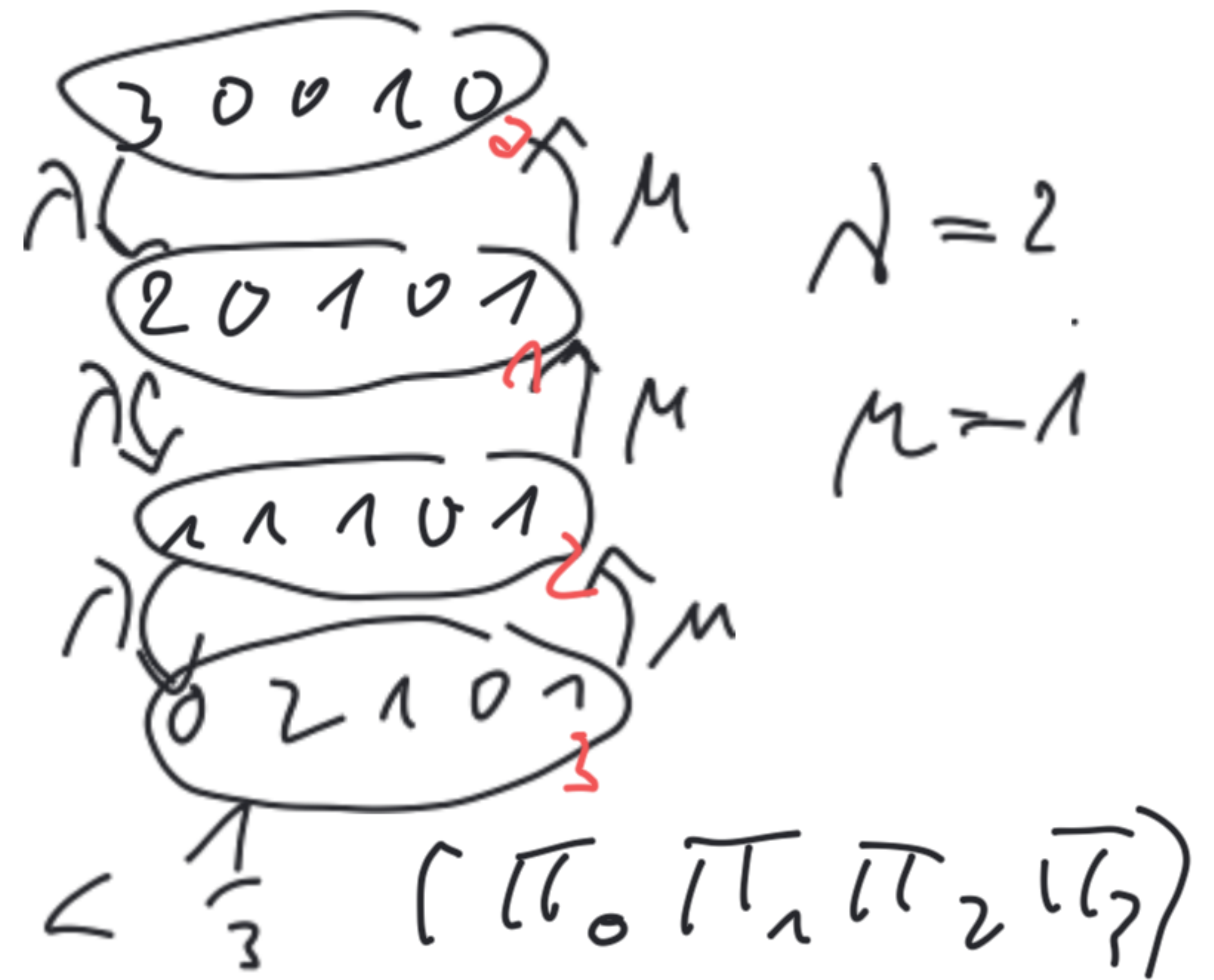
$$\begin{bmatrix} -2 & 2 & 0 & 0 \\ 1 & -3 & 2 & 0 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\Delta < \frac{1}{3}$$

$$\Rightarrow \frac{1}{8}$$

$$\pi_0 + \pi_1$$

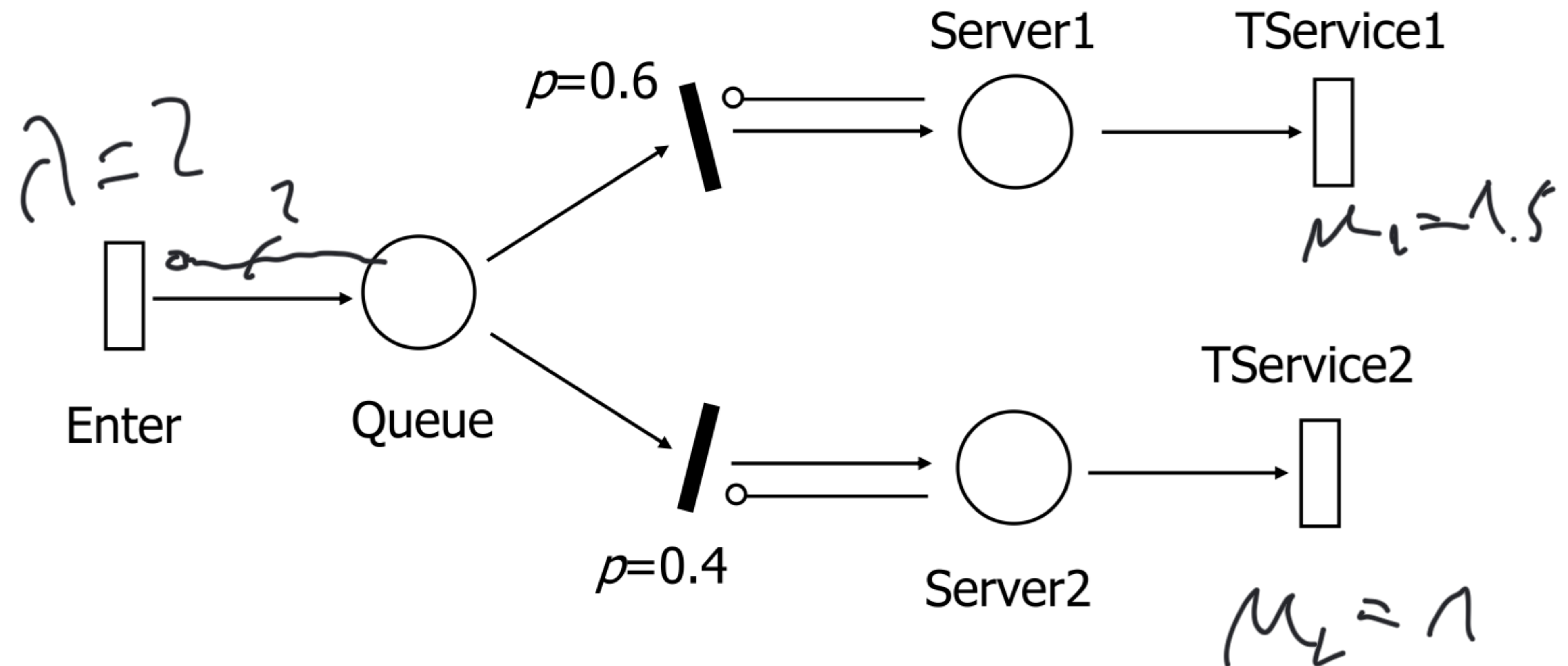
$$\pi_2 \cdot 1 + \pi_3 \cdot 2$$



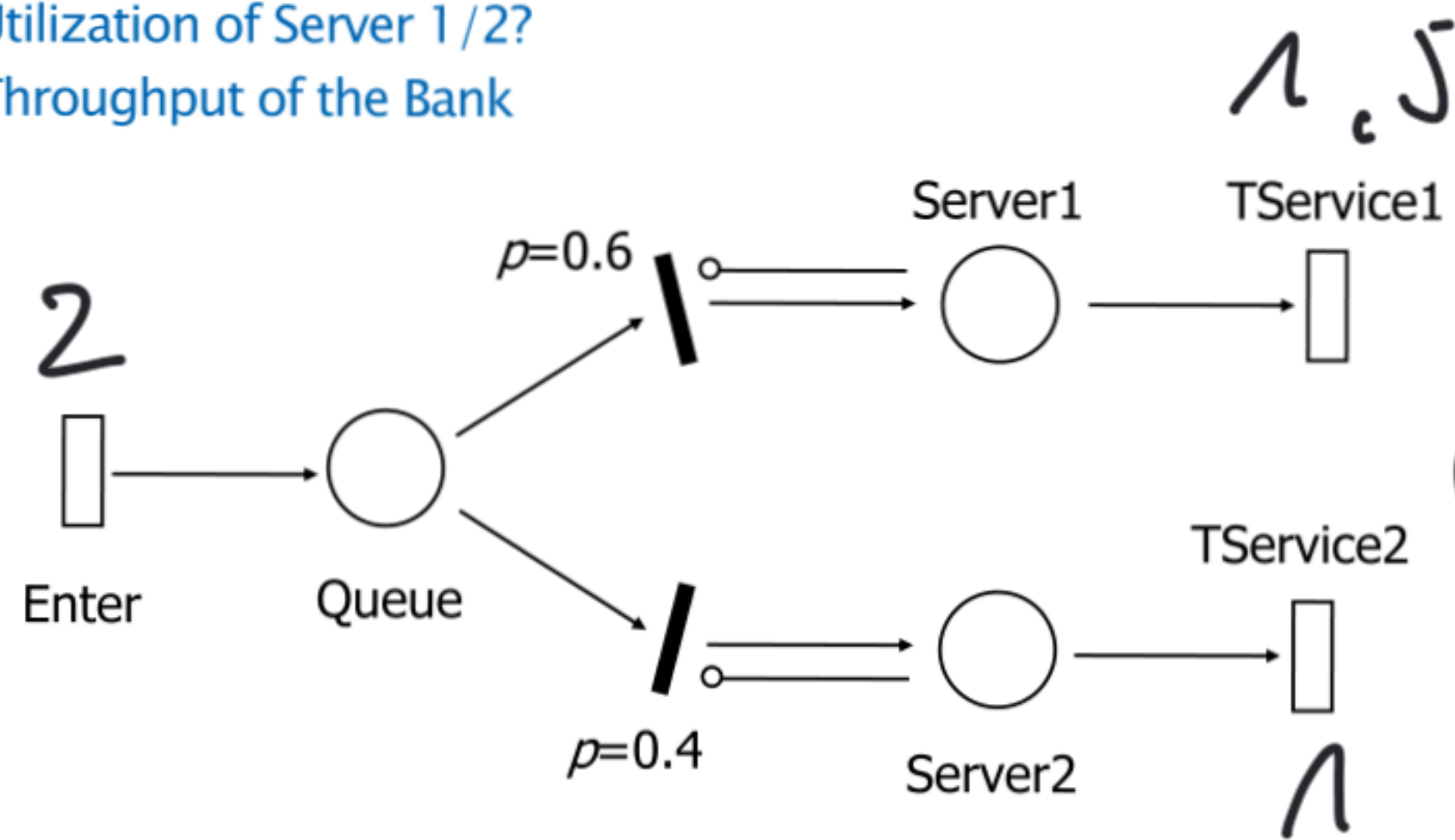
Example: The Bank

Utilization of Server 1 / 2?

Throughput of the Bank



Utilization of Server 1 / 2?
Throughput of the Bank



$$\begin{bmatrix} -2 & 1.2 & 0.8 & 0 & 0 & 0 \\ 1.5 & -3.5 & 0 & 2 & 0 & 0 \\ 1 & 0 & -3 & 2 & 0 & 0 \\ 0 & 1 & 1.5 & -4.5 & 2 & 0 \\ 0 & 0 & 0 & 2.5 & -4.5 & 2 \\ 0 & 0 & 0 & 0 & 2.5 & -2.5 \end{bmatrix}$$

$$\overline{\pi}_1 + \overline{\pi}_3 + \overline{\pi}_4 + \overline{\pi}_5$$

$$\overline{\pi}_2 + \overline{\pi}_7 + \overline{\pi}_4 + \overline{\pi}_5$$

$$U_1 \cdot \mu_1 + U_2 \cdot \mu_2$$

$$\Delta = 0.1$$