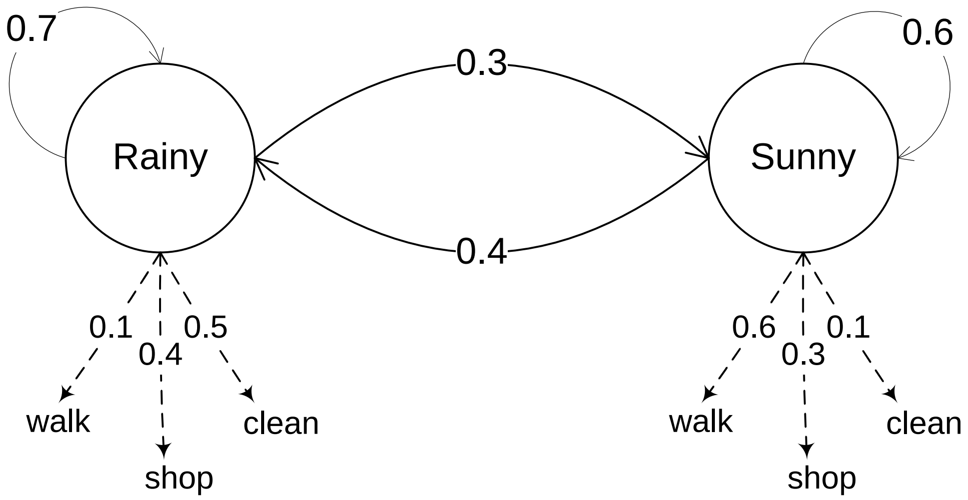
ADM - Hidden Markov Models

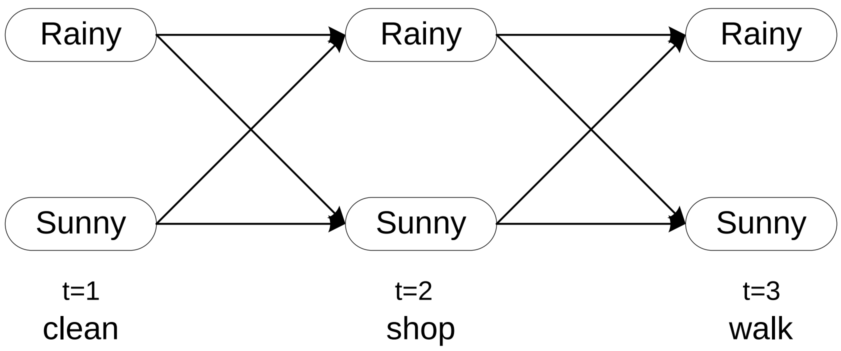


$(S,V,A,B,\Pi) = (\{Rainy,Sunny\},\{walk,shop,clean\},A,B,(0.6,0.4))$

$$A = \begin{array}{c} \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cc} R & S \end{array} \\ \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ R \\ S \end{array} \begin{array}{c} \begin{array}{ccc} w & s & c \end{array} \\ \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.6 & 0.3 & 0.1 \end{bmatrix} \end{array}$$

$$O = (clean, shop, walk)$$



Evaluation

$$P(csw/RRR) = \pi_R b_{Rc} a_{RR} b_{Rs} a_{RR} b_{Rw} = 0.6*0.5*0.7*0.4*0.7*0.1 = 0.00588$$

$$P(csw/RRS) = \pi_R b_R c a_{RR} b_{Rs} a_{RS} b_{Sw} = 0.4*0.1*0.6*0.3*0.6*0.6 = 0.01512$$

$$\vdots$$

$$P(csw) = \sum P(csw/xxx)$$

Forward Algorithm

$$\alpha_1(R) = \pi_R b_{Rc} = 0.6*0.5 = 0.3$$

$$\alpha_1(S) = \pi_S b_{Sc} = 0.4*0.1 = 0.04$$

$$\alpha_2(R) = \alpha_1(R) a_{RR} b_{Rs} + \alpha_1(S) a_{SR} b_{Rs} = 0.3*0.7*0.4 + 0.04*0.4*0.4 = 0.0904$$

$$\alpha_2(S) = \alpha_1(R) a_{RS} b_{Ss} + \alpha_1(S) a_{SS} b_{Ss} = 0.3*0.3*0.3 + 0.04*0.6*0.3 = 0.0342$$

$$\alpha_3(R) = \alpha_2(R) a_{RR} b_{Rw} + \alpha_2(S) a_{SR} b_{Rw} = 0.0904*0.7*0.1 + 0.0342*0.4*0.1 = 0.007696$$

$$\alpha_3(S) = \alpha_2(R) a_{RS} b_{Sw} + \alpha_2(S) a_{SS} b_{Sw} = 0.0904*0.3*0.6 + 0.0342*0.6*0.6 = 0.028584$$

$$P(csw) = \alpha_3(R) + \alpha_3(S) = 0.03628$$

Decoding

$$P(csw/RRR) = \pi_R b_{Rc} a_{RR} b_{Rs} a_{RR} b_{Rw} = 0.6*0.5*0.7*0.4*0.7*0.1 = 0.00588$$

$$P(csw/RRS) = \pi_R b_{Rc} c a_{RR} b_{Rs} a_{RS} b_{Sw} = 0.6*0.5*0.7*0.4*0.3*0.6 = 0.01512$$

$$\vdots$$

$$Path = \arg\max_{xxx} \{P(csw/xxx)\}$$

Viterbi Algorithm

$$\delta_1(R) = \pi_R b_{Rc} = 0.6*0.5 = 0.3$$

$$\delta_1(S) = \pi_S b_{Sc} = 0.4*0.1 = 0.04$$

$$\delta_2(R) = \max\{\delta_1(R) a_{RR} b_{Rs}, \delta_1(S) a_{SR} b_{Rs}\} = \max\{0.084, 0.0064\} = 0.084$$

$$\delta_2(S) = \max\{\delta_1(R) a_{RS} b_{Ss}, \delta_1(S) a_{SS} b_{Ss}\} = \max\{0.027, 0.0072\} = 0.027$$

$$\delta_3(R) = \max\{\delta_2(R) a_{RR} b_{Rw}, \delta_2(S) a_{SR} b_{Rw}\} = \max\{0.00588, 0.00108\} = 0.00588$$

$$\delta_3(S) = \max\{\delta_2(R) a_{RS} b_{Sw}, \delta_2(S) a_{SS} b_{Sw}\} = \max\{0.01512, 0.00972\} = 0.01512$$

$$\max\{\delta_3(R), \delta_3(S)\} = 0.01512 = P(csw/RRS)$$