

CTMCs

discretization

$$P = I + dt * Q$$

$$dt < 1 / \max |q_{ij}|$$

$$p_i * Q = 0 \rightarrow p_i = p_i * P$$

examples

- amoeba - birth-death-process
- 4 machines, 1 repair person
- two machines, two repair persons

combining rates

machine shop example

- balance equation
- steady state
- $dp_i/dt = p_i * Q$
- transient behaviour
- watertank analogy
- properties
- example
- probability vector
- properties
- example
- infinitesimal generator matrix
- failure rate λ
- repair rate μ
- system description

Satellite Example

- System description
- exponential lifetime
- derived dtmc for time period 1, 10, 2 ...
- $P(t) = ((1,0)(1 - e^{-\lambda t}, e^{-\lambda t}))$

Markov Property

- memoryless
- $P(X(t+dt)=x | X(s)=s, s \leq t) = P(X(t+dt)=x | X(t)=x(t))$
- P(event in interval) is independent of waiting time
- $P(T > t+dt | T > t) = P(T > dt)$
- state holding time is exponential
- valid only for exp distribution

fluid flow rates

- $p_{10}(1)/1 \dots p_{10}(0.5)/0.5$
- $\lim(p_{ij}(t) - p_{ij}(0)/t) = \lim(p_{ij}(t)/t) = q_{ij}$
- $\lim(p_{ii}(t) - p_{ii}(0)/t) = \lim(p_{ii}(t) - 1/t) = q_{ii}$
- water tank analogy
 - p_i = fluid level
 - λ = pipe cross section
 - flow = change in probability
- graphical representation

Application areas

- logistics
- safety, reliability
- manufacturing
- traffic
- computer networks
- ultimately, every ds model