Total Enumeration

$$P(csw/RRR) = \pi_R b_{Rc} a_{RR} b_{Rs} a_{RR} b_{Rw} = 0.6*0.5*0.7*0.4*0.7*0.1 = 0.00588$$
$$P(csw/RRS) = \pi_R b_{Rc} a_{RR} b_{Rs} a_{RS} b_{Sw} = 0.6*0.5*0.7*0.4*0.3*0.6 = 0.01512$$
$$P(csw/RSR) = \pi_R b_{Rc} a_{RS} b_{Ss} a_{SR} b_{Rw} = 0.6*0.5*0.3*0.3*0.4*0.1 = 0.001080$$
$$P(csw/RSS) = \pi_R b_{Rc} a_{RS} b_{Ss} a_{SS} b_{Sw} = 0.6*0.5*0.3*0.3*0.6*0.6 = 0.00972$$
$$P(csw/SRR) = \pi_S b_{Sc} a_{SR} b_{Rs} a_{RR} b_{Rw} = 0.4*0.1*0.4*0.4*0.7*0.1 = 0.000448$$
$$P(csw/SRS) = \pi_S b_{Sc} a_{SR} b_{Rs} a_{RS} b_{Sw} = 0.4*0.1*0.4*0.4*0.3*0.6 = 0.001152$$
$$P(csw/SSR) = \pi_S b_{Sc} a_{SS} b_{Ss} a_{SR} b_{Rw} = 0.4*0.1*0.6*0.3*0.4*0.1 = 0.000288$$
$$P(csw/SSS) = \pi_S b_{Sc} a_{SS} b_{Ss} a_{SS} b_{Sw} = 0.4*0.1*0.6*0.3*0.6*0.6 = 0.002592$$
$$P(csw) = \sum P(csw/xxx) = 0.03628$$

$$Path = \arg\max_{xxx}\{P(csw/xxx)\} = RRS$$

Evaluation

Forward Algorithm

$$\alpha_1(R) = \pi_R b_{Rc} = 0.6*0.5 = 0.3$$
$$\alpha_1(S) = \pi_S b_{Sc} = 0.4*0.1 = 0.04$$
$$\alpha_2(R) = \alpha_1(R) a_{RR} b_{Rs} + \alpha_1(S) a_{SR} b_{Rs} = 0.3*0.7*0.4 + 0.04*0.4*0.4 = 0.0904$$
$$\alpha_2(S) = \alpha_1(R) a_{RS} b_{Ss} + \alpha_1(S) a_{SS} b_{Ss} = 0.3*0.3*0.3 + 0.04*0.6*0.3 = 0.0342$$
$$\alpha_3(R) = \alpha_2(R) a_{RR} b_{Rw} + \alpha_2(S) a_{SR} b_{Rw} = 0.0904*0.7*0.1 + 0.0342*0.4*0.1 = 0.007696$$
$$\alpha_3(S) = \alpha_2(R) a_{RS} b_{Sw} + \alpha_2(S) a_{SS} b_{Sw} = 0.0904*0.3*0.6 + 0.0342*0.6*0.6 = 0.028584$$
$$P(csw) = \alpha_3(R) + \alpha_3(S) = 0.03628$$

Decoding

Viterbi Algorithm

$$\delta_1(R) = \pi_R b_{Rc} = 0.6*0.5 = 0.3$$
$$\delta_1(S) = \pi_S b_{Sc} = 0.4*0.1 = 0.04$$
$$\delta_2(R) = \max\{\delta_1(R) a_{RR} b_{Rs}, \delta_1(S) a_{SR} b_{Rs}\} = \max\{0.084, 0.0064\} = 0.084$$
$$\delta_2(S) = \max\{\delta_1(R) a_{RS} b_{Ss}, \delta_1(S) a_{SS} b_{Ss}\} = \max\{0.027, 0.0072\} = 0.027$$
$$\delta_3(R) = \max\{\delta_2(R) a_{RR} b_{Rw}, \delta_2(S) a_{SR} b_{Rw}\} = \max\{0.00588, 0.00108\} = 0.00588$$
$$\delta_3(S) = \max\{\delta_2(R) a_{RS} b_{Sw}, \delta_2(S) a_{SS} b_{Sw}\} = \max\{0.01512, 0.00972\} = 0.01512$$
$$\max\{\delta_3(R), \delta_3(S)\} = 0.01512 = P(csw/RRS)$$